Original Article

Support Neighbourly Edge Irregular Fuzzy Graphs

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Abstract

In this paper, a new class of edge irregular fuzzy graphs called support neighbourly edge irregular fuzzy graphs and support neighbourly edge totally irregular fuzzy graphs were introduced. Also we proved some conditions under which the graphs are both support neighbourly edge irregular fuzzy graphs and support neighbourly edge totally irregular fuzzy graphs and analyse the properties of some special graphs through the nature of edge membership values.

Keywords: support, support of an edge, total support of an edge, support neighbourly edge irregular fuzzy graphs, support neighbourly edge totally irregular fuzzy graphs.

AMS subject classification : Primary: 05C12, Secondary: 05C72.

1 INTRODUCTION

Azriel Rosenfeld [14] made a strong basis for the fuzzy graph theory after Lofti.A.Zadeh [15] in 1975. Now, it was the fast growing and having numerous application in Graph Theory. Neighbourly irregular graphs was introduced by Gnaana Bhragsam and Ayyasamy[3]. But the same concept was introduced in fuzzy graphs by A. Nagoorgani and S.R. Latha. The concept of edge degree in fuzzy graphs was introduced by N.Kumaravel and K.Radha[12]. Support of an edge has been introduced by K. Amutha and N.R. Santhi Maheswari[1].R. Muneeswari and N.R. Santhi Maheswari[4] introduced the support of an edge and studied the properties of support edge regular fuzzy graphs. These makes me to work with the support neighbourly edge irregular fuzzy graphs by combining support of an edge and neighbourly irregular fuzzy graphs and studied its properties through some special graphs, such as barbell graph, path, cycles, star with some specific edge membership function.

2 PRELIMINARIES

In this section, we recall the notions related to fuzzy graphs.

Definition 2.1. *[14] A fuzzy graph is a pair of functions $G : (\sigma, \mu)$, where $\sigma : V \rightarrow [0,1]$ is a fuzzy subset of a non-empty set V and $\mu : V \times V \rightarrow [0,1]$ is a symmetric fuzzy relation on σ such that $\mu(uv) \leq \sigma(u) \Lambda \sigma(v), \forall u, v \in V$ where uv denote the edge between u and v and $\sigma(u) \Lambda \sigma(v)$ denotes the minimum of $\sigma(u)$ and $\sigma(v)$, σ is called the fuzzy vertex set of V and μ is called the fuzzy edge set of E.*

Definition 2.2. *[7] Let $G : (\sigma, \mu)$ be a connected fuzzy graph on $G^*(V, E)$. The degree of a vertex u is $d_G(u) = \sum_{u \neq v} \mu(uv)$, for $uv \in E$ and $\mu(uv) = 0$ for $uv \notin E$; this is equivalent to $d_G(u) = \sum_{uv \in E} \mu(uv)$.*

Definition 2.3. *[6] Let $G : (\sigma, \mu)$ be a connected fuzzy graph on $G^*(V, E)$. If $d_G(v) = k$ for all $v \in V$, then G is said to be regular fuzzy graph of degree k.*

Definition 2.4. *[13] Let $G : (\sigma, \mu)$ be a connected fuzzy graph on $G^*(V, E)$. Then G is said to be an irregular fuzzy graph if there exists a vertex which is adjacent to the vertices with distinct degrees.*

Definition 2.5. *[13] Let $G : (\sigma,\mu)$ be a connected fuzzy graph on $G^*(V,E)$. Then G is said to be a totally irregular fuzzy graph if there exists a vertex which is adjacent to the vertices with distinct total degrees.*

Definition 2.6. *[5] Let $G : (\sigma,\mu)$ be a connected fuzzy graph on $G^*(V,E)$. Then G is said to be a neighbourly irregular fuzzy graph if every pair of adjacent vertices have distinct degrees.*

Definition 2.7. *[5] Let $G : (\sigma,\mu)$ be a connected fuzzy graph on $G^*(V,E)$. Then G is said to be a neighbourly totally irregular fuzzy graph if every pair of adjacent vertices have distinct total degrees.*

Definition 2.8. *[2] The degree of an edge uv in the underlying graph is defined as $d_{G*}(uv) = d_{G*}(u) + d_{G*}(v) - 2$.*

Definition 2.9. *[12] Let $G : (\sigma,\mu)$ be a connected fuzzy graph on $G^*(V,E)$. The degree of an edge uv is defined as $d_G(uv) = d_G(u) + d_G(v) - 2\mu(uv)$.*
The minimum degree of an edge is $\delta_E(G) = \wedge\{d_G(uv)/uv \in E\}$
The maximum degree of an edge is $\Delta_E(G) = \vee\{d_G(uv)/uv \in E\}$.

Definition 2.10. *[12] Let $G : (\sigma,\mu)$ be a connected fuzzy graph on $G^*(V,E)$. The total degree of an edge uv is defined as $td_G(uv) = d_G(u) + d_G(v) - \mu(uv) = d_G(uv) + \mu(uv)$.*
The minimum total degree of an edge is $\delta_{tE}(G) = \wedge\{td_G(uv)/uv \in E\}$
The maximum total degree of an edge is $\Delta_{tE}(G) = \vee\{td_G(uv)/uv \in E\}$.

Definition 2.11. *[10] Let $G : (\sigma,\mu)$ be a connected fuzzy graph on $G^*(V,E)$. Then G is said to be a neighbourly edge irregular fuzzy graph if every pair of adjacent edges have the distinct degrees.*

Definition 2.12. *[10] Let $G : (\sigma,\mu)$ be a connected fuzzy graph on $G^*(V,E)$. Then G is said to be a neighbourly edge totally irregular fuzzy graph if every pair of adjacent edges have the distinct total degrees.*

Definition 2.13. *[1] The support $s(e)$ of an edge e in the underlying graph is the sum of edge degrees of its neighbour edges. That is, $s(e) = \sum_{e_i \in N(e)} d(e_i)$.*

Definition 2.14. *[4] Let $G : (\sigma,\mu)$ be a connected fuzzy graph on $G^*(V,E)$. The support $s_G(e)$ of an edge e is the sum of edge degrees of its neighbour edges. That is, $s_G(e) = \sum_{e_i \in N(e)} d_G(e_i)$.*

Definition 2.15. *[4] Let $G : (\sigma,\mu)$ be a connected fuzzy graph on $G^*(V,E)$. The total support $ts_G(e)$ of an edge e is $ts_G(e) = \sum_{e_i \in N(e)} d_G(e_i) + \mu(e) = s_G(e) + \mu(e)$.*

3 SUPPORT NEIGHBOURLY EDGE IRREGULAR FUZZY GRAPHS AND SUPPORT NEIGHBOURLY EDGE TOTALLY IRREGULAR FUZZY GRAPHS

In this section we analyse the relation between support neighbourly edge irregular fuzzy graphs and support neighbourly edge totally irregular fuzzy graph.

Definition 3.1. *Let $G : (\sigma,\mu)$ be a fuzzy graph on $G^*(V,E)$. Then G is said to be support neighbourly edge irregular fuzzy graph if every pair of adjacent edges of G has distinct supports.*

Example 3.2. *The following graph shows the existence of support neighbourly edge irregular fuzzy graph. Let $G : (\sigma,\mu)$ be a connected fuzzy graph on $G^*(V,E)$, a butterfly graph.*

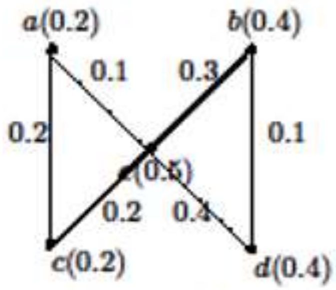

Figure 1

From Figure 1, $d_G(a) = 0.3, d_G(b) = 0.4, d_G(c) = 0.4, d_G(d) = 0.5, d_G(e) = 1.0$ *and* $d_G(ac) = 0.3,\ d_G(ae) = 1.1,\ d_G(ce) = 1.0, d_G(be) = 0.8, d_G(de) = 0.7, d_G(bd) = 0.7$.
Support of the edges are calculated below. $s_G(ac) = d_G(ae) + d_G(ce) = 1.1 + 1.0 = 2.1$ *Similarly,* $s_G(ae) = 2.8, s_G(ce) = 2.9, s_G(bd) = 1.5, s_G(be) = 3.5, s_G(de) = 3.6$. *Here every pair of adjacent edges has distinct supports. Hence* G *is support neighbourly edge irregular fuzzy graph.*

Definition 3.3.*Let* $G : (\sigma, \mu)$ *be a fuzzy graph on* $G^*(V, E)$. *Then* G *is said to be support neighbourly edge totally irregular fuzzy graph if any two adjacent edges of* G *have distinct total supports.*

Example 3.4.*The following graph shows the existence of support neighbourly edge totally irregular fuzzy graph. Let* $G : (\sigma, \mu)$ *be a connected fuzzy graph on* $G^*(V, E)$.

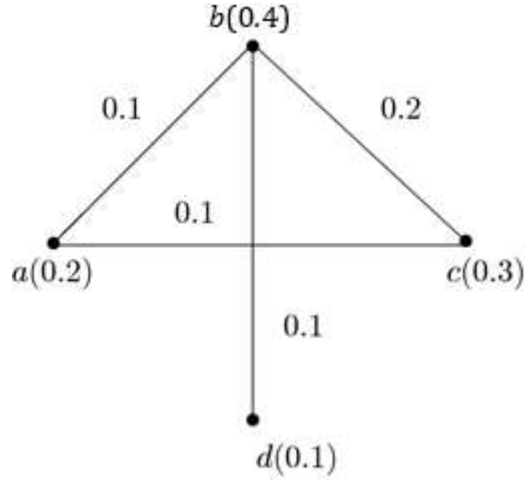

Figure 2

From Figure 2, $d_G(a) = 0.2, d_G(b) = 0.4, d_G(c) = 0.3, d_G(d) = 0.1$ *and* $d_G(ab) = 0.4, d_G(ac) = 0.3, d_G(bc) = 0.3, d_G(bd) = 0.3$.
Support and total support of the edges are as follows.
$s_G(ab) = 0.9, s_G(ac) = 0.7, s_G(bc) = 1.0, s_G(bd) = 0.7$ *and* $ts_G(ab) = 1.0, ts_G(ac) = 0.8, ts_G(bc) = 1.2, ts_G(bd) = 0.8$. *Here every pair of adjacent edges have distinct total supports. Hence* G *is support neighbourly edge totally irregular fuzzy graph.*

Remark 3.5.*Let* $G : (\sigma, \mu)$ *be a connected fuzzy graph on* $G^*(V, E)$. *If* G *is a support neighbourly edge irregular fuzzy graph, then* G *need not be a support neighbourly edge totally irregular fuzzy graph.*

Example 3.6.*Consider a connected fuzzy graph* $G : (\sigma, \mu)$ *on* $G^*(V, E)$.

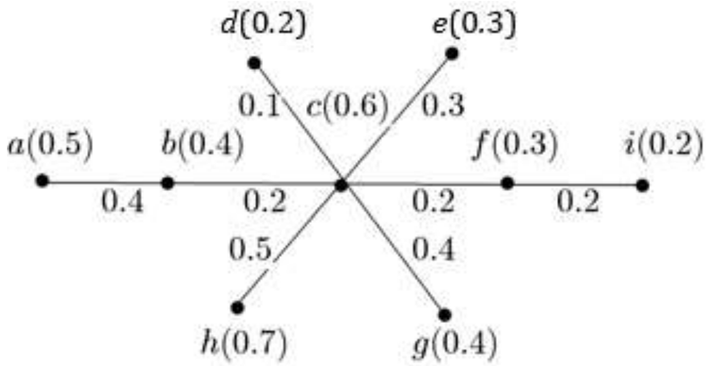

Figure 3

From Figure 3, $d_G(a) = 0.4, d_G(b) = 0.6, d_G(c) = 1.7, d_G(d) = 0.1, d_G(e) = 0.3, d_G(f) = 0.4, d_G(g) = 0.4, d_G(h) = 0.5, d_G(i) = 0.2$ *and* $d_G(ab) = 0.2,\ d_G(bc) = 1.9, d_G(cd) = 1.6, d_G(ec) = 1.4, d_G(cf) = 1.7, d_G(cg) = 1.3,\ d_G(ch) = 1.2, d_G(fi) = 0.2$.
Support and total support of the edges are as follows.
$s_G(ab) = 1.6, s_G(bc) = 7.4, s_G(cd) = 7.5, s_G(ec) = 7.7, s_G(cf) = 7.6, s_G(cg) = 7.8, s_G(ch) = 7.9, s_G(fi) = 1.7$ *and* $ts_G(ab) = 2.0, ts_G(bc) = 7.6, ts_G(cd) = 7.6, ts_G(ec) = 8.0, ts_G(cf) = 7.8, ts_G(cg) = 8.2, ts_G(ch) = 8.4, ts_G(fi) = 1.9$. *Here every pair of adjacent edges of* G *have distinct supports. Thus* G *is support neighbourly edge*

irregular fuzzy graph. But the pair of adjacent edges bc and cd have same total support 7.6. *Hence G is not support neighbourly edge totally irregular fuzzy graph.*

Remark 3.7. *Consider a connected fuzzy graph $G : (\sigma,\mu)$ on $G^*(V,E)$. If G is a support neighbourly edge totally irregular fuzzy graph, then G need not be a support neighbourly edge irregular fuzzy graph.*

Example 3.8. *Consider a connected fuzzy graph $G : (\sigma,\mu)$ on $G^*(V,E)$.*

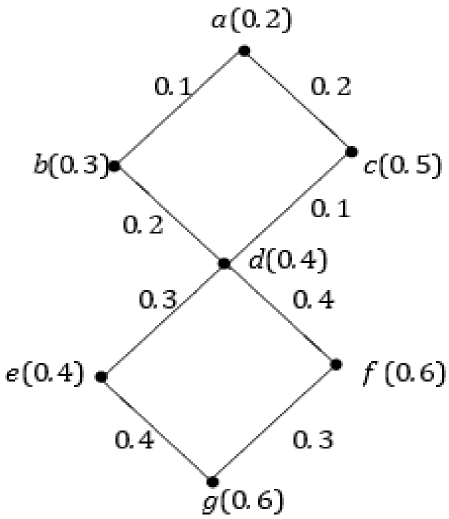

Figure 4

From the above Figure, $d_G(a) = d_G(b) = d_G(c) = 0.3, d_G(d) = 1.0, d_G(e) = d_G(f) = d_G(g) = 0.7$ *and* $d_G(ab) = 0.4, d_G(ac) = 0.2, d_G(bd) = 0.9, d_G(cd) = 1.1, d_G(de) = 1.1, d_G(df) = 0.9, d_G(eg) = 0.6, d_G(fg) = 0.8$.

Support and total support of the edges are as follows.

$s_G(ab) = 1.1, s_G(ac) = 1.5, s_G(bd) = 3.5, s_G(cd) = 3.1, s_G(de) = 3.5, s_G(df) = 3.9, s_G(eg) = 1.9, s_G(fg) = 1.5$ *and* $ts_G(ab) = 1.2, ts_G(ac) = 1.7, ts_G(bd) = 3.7, ts_G(cd) = 3.2, ts_G(de) = 3.8, ts_G(df) = 4.3, ts_G(eg) = 2.3, ts_G(fg) = 1.8$. *Here the pair of adjacent edges bd and de have same support* 3.5. *Hence G is not support neighbourly edge irregular fuzzy graph. But every pair of adjacent edges of G have distinct total supports. Thus G is support neighbourly edge totally irregular fuzzy graph.*

Remark 3.9. *There exist some graphs which are both support neighbourly edge irregular fuzzy graph and support neighbourly edge totally irregular fuzzy graph.*

Example 3.10. *Consider a connected fuzzy graph $G : (\sigma,\mu)$ on $G^*(V,E)$, a barbell graph B*(4,4).

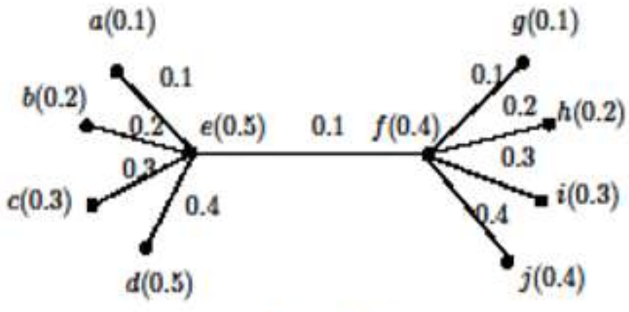

Figure 5

FromFigure5, $d_G(a) = 0.1, d_G(b) = 0.2, d_G(c) = 0.3, d_G(d) = 0.4, d_G(e) = 1.1, d_G(f) = 1.1, d_G(g) = 0.1, d_G(h) = 0.2, d_G(i) = 0.3, d_G(j) = 0.4$ *and* $d_G(ae) = 1.0, d_G(be) = 0.9, d_G(ce) = 0.8, d_G(de) = 0.7, d_G(ef) = 2.0, d_G(gf) = 1.0, d_G(hf) = 0.9, d_G(if) = 0.8, d_G(jf) = 0.7$.

Support and total support of the edges are as follows. $s_G(ae) = 4.4, s_G(be) = 4.5, s_G(ce) = 4.6, s_G(de) = 4.7, s_G(ef) = 6.8, s_G(gf) = 4.4, s_G(hf) = 4.5, s_G(if) = 4.6, s_G(jf) = 4.7$ *and* $ts_G(ae) = 4.5, ts_G(be) = 4.7, ts_G(ce) = 4.9, ts_G(de) = 5.1, ts_G(ef) = 6.9, ts_G(gf) = 4.5, ts_G(hf) = 4.7, ts_G(if) = 4.9, ts_G(jf) = 5.1$.

Here every pair of adjacent edges have distinct supports and distinct total supports. Hence G is both support neighbourly edge irregular fuzzy graph and support neighbourly edge totally irregular fuzzy graph

Remark 3.11. *There exist some graphs which are neither support neighbourly edge irregular fuzzy graph nor support neighbourly edge totally irregular fuzzy graph.*

Example 3.12. *Consider a connected fuzzy graph $G : (\sigma, \mu)$ on $G^*(V, E)$, a star graph $K_{1,6}$.*

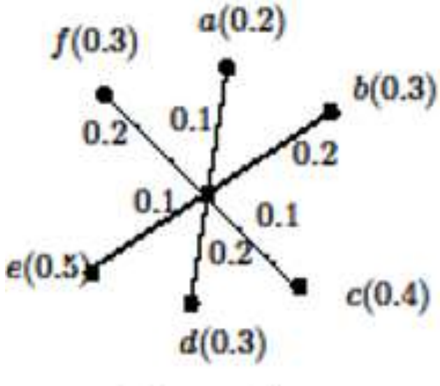

Figure 6

From Figure 6, $d_G(a) = 0.1, d_G(b) = 0.2, d_G(c) = 0.1, d_G(d) = 0.2, d_G(e) = 0.1, d_G(f) = 0.2$ *and* $d_G(ag) = 0.8, d_G(bg) = 0.7, d_G(cg) = 0.8, d_G(dg) = 0.7, d_G(eg) = 0.8, d_G(fg) = 0.7$.

Support and total support of the edges are as follows.
$s_G(ag) = 3.7, s_G(bg) = 3.8$, $s_G(cg) = 3.7$, $s_G(dg) = 3.8$, $s_G(eg) = 3.7$, $s_G(fg) = 3.8$ *and* $ts_G(ag) = 3.8, ts_G(bg) = 4.0, ts_G(cg) = 3.8, ts_G(dg) = 4.0, ts_G(eg) = 3.8, ts_G(fg) = 4.0$. *Here some pair of adjacent edges have same support and same total support. Hence G is neither support neighbourly edge irregular fuzzy graph nor support neighbourly edge totally irregular fuzzy graph.*

Theorem 3.13. *Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^*(V,E)$, a support neighbourly edge irregular fuzzy graph. If μ is constant, then G is both support neighbourly edge irregular fuzzy graph and support neighbourly edge totally irregular fuzzy graph.*

Proof. Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^*(V,E)$, a support neighbourly edge irregular fuzzy graph. Suppose μ is constant. Let $\mu(ei) = r\ \forall i$. Then $s_G(e_i) = rs_{G*}(e_i)\ \forall\ e_i \in E(G)$. Since G is support neighbourly edge irregular fuzzy graph, $s_{G*}(e_i) \neq s_{G*}(e_j)$, if e_I, e_J are adjacent $\Rightarrow rs_{G*}(e_i) \neq rs_{G*}(e_j)$, if e_I, e_J are adjacent $\Rightarrow s_G(e_i) \neq s_G(e_j)$, if e_I, e_J are adjacent $\Rightarrow s_G(e_i) + \mu(e_i) \neq s_G(e_j) + \mu(e_j)$, if e_I, e_J are adjacent $\Rightarrow ts_G(e_i) \neq ts_G(e_j)$, if e_I, e_J are adjacent. Here every pair of edges of G have distinct supports and distinct total supports. Thus G is both support neighbourly edge irregular fuzzy graph and Support neighbourly edge totally irregular fuzzy graph.

Remark 3.14. *The converse of the above theorem need not be true.*

Example 3.15. *The following example illustrates the above remark. Consider the connected fuzzy graph $G : (\sigma, \mu)$ on $G^*(V, E)$*

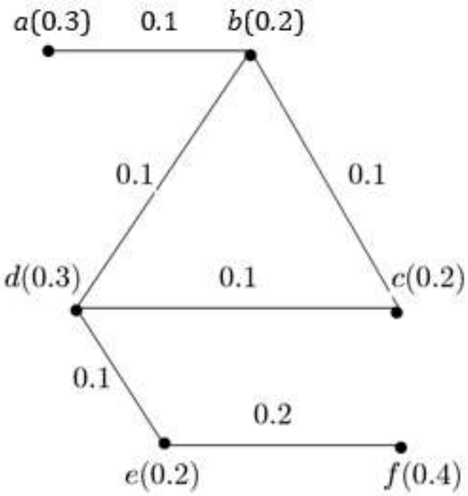

Figure 7

From above Figure , $d_{G*}(a) = 1, d_{G*}(b) = 3, d_{G*}(c) = 2, d_{G*}(d) = 3, d_{G*}(e) = 2, d_{G*}(f) = 1$ *and* $d_{G*}(ab) = 2, d_{G*}(bc) = 3, d_{G*}(cd) = 3, d_{G*}(bd) = 4, d_{G*}(de) = 3, d_{G*}(ef) = 1$. *Support of the edges in the underlying graphs are* $s_{G*}(ab) = 7, s_{G*}(bc) = 9, s_{G*}(cd) = 10, s_{G*}(bd) = 11, s_{G*}(de) = 8, s_{G*}(ef) = 3$. *So no two adjacent edges have same support. Thus* G^**is support neighbourly edge irregular graph.*

For the fuzzy graph $G, d_G(a) = 0.1, d_G(b) = 0.3, d_G(c) = 0.2, d_G(d) = 0.3, d_G(e) = 0.3, d_G(f) = 0.2$, *and* $d_G(ab) = 0.2, d_G(bc) = 0.3, d_G(cd) = 0.3, d_G(bd) = 0.4, d_G(de) = 0.4, d_G(ef) = 0.1$.
Support and total support of the edges are as follows. $s_G(ab) = 0.7, s_G(bc) = 0.9, s_G(cd) = 1.1, s_G(bd) = 1.2, s_G(de) = 0.8, s_G(ef) = 0.4$ *and* $ts_G(ab) = 0.8, ts_G(bc) = 1.0, ts_G(cd) = 1.2, ts_G(bd) = 1.3, ts_G(de) = 0.9, ts_G(ef) = 0.6$. *Here every pair of adjacent edges have distinct supports and distinct total supports. Hence G is both support neighbourly edge irregular fuzzy graph and support neighbourly edge totally irregular fuzzy graph. But μ is not constant.*

Theorem 3.16.*Let* $G : (\sigma, \mu)$*be a fuzzy graph on* $G^*(V, E)$, *a r-regular graph. If r distinct values are taken as the membership values of the edges in such a way that no two adjacent edges have same membership values, then G is support neighbourly edge irregular fuzzy graph.*
Proof. Let $G : (\sigma, \mu)$*be a fuzzy graph on* $G^*(V, E)$,a r-regular graph. Suppose the edges takes the $r$distinct values in such a way that no two adjacent edges have the same values. Let the $r$values be $m, 2m, 3m, \ldots, rm$. Then clearly $r$edges incident on each vertex has the values $m, 2m, 3m, \ldots, rm$. So $d_G(u_i) = \frac{r(r+1)m}{2}$.The edges with the membership values $m, 2m, 3m, \ldots, rm$ has the degree $r(r+1)m - 2m, r(r+1)m - 4m, \ldots, r(r+1)m - 2rm$ respectively. Here each edge is incident on $2(r-1)$ edges (*i.e.* 2 sets of $r-1$ edges.) The support of the edge with membership value i is $s_G(u_i) = 2\sum_{j=1}^{r}(r(r+1)m - 2jm) - 2r(r+1)m + 4im = 2r^2(r+1)m - 2r(r+1)m - 2r(r+1)m + 4im = 2(r-2)r(r+m)m + 4im$. Hence no two adjacent edges have same support. Thus $G$is support neighbourly edge irregular fuzzy graph.

4 SUPPORT NEIGHBOURLY EDGE IRREGULARITY FOR SOME SPECIAL GRAPHS UNDER SPECIFIC EDGE MEMBERSHIP FUNCTION

Theorem 4.1.*Let* $G : (\sigma, \mu)$*be a fuzzy graph on* $G^*(V, E)$, *an even cycle* $C_n, n = 2k$. *If the alternate edges have same membership values, then G is support neighbourly edge irregular fuzzy graph.*

Proof. Let $G : (\sigma, \mu)$*be a fuzzy graph on* $G^*(V, E)$, *an even cycle* $C_n, n = 2k$. Suppose the alternate edges have same membership values. Let $a_1, a_2, \ldots, a_n$be the vertices of G and $a_i a_{i+1} = e_i, a_n a_1 = e_n$. Let $\mu(e_i) = \begin{cases} r \text{ if } i \text{ is odd} \\ 2r \text{ if } i \text{ is even.} \end{cases}$
Then $d_G(v_i) = 3r, \forall i$. $d_G(e_i) = \begin{cases} 4r \text{ if } i \text{ is odd} \\ 2r \text{ if } i \text{ is even} \end{cases}$ and $s_G(e_i) = d_G(e_{i-1}) + d_G(e_{i+1}) = \begin{cases} 4r \text{ if } i \text{ is odd} \\ 8r \text{ if } i \text{ is even.} \end{cases}$

Thus no two adjacent edges have same support. Hence G is support neighbourly edge irregular fuzzy graph.

Theorem 4.2.*Let* $G : (\sigma, \mu)$*be a fuzzy graph on* $G^*(V, E)$, *a star graph* $K_{1,n}$. *Then G is support neighbourly edge irregular fuzzy graph iff no two adjacent edges of G have same membership values.*

Proof. Let $G : (\sigma, \mu)$*be a fuzzy graph on* $G^*(V, E)$, *a star graph* $K_{1,n}$. Suppose G is support neighbourly edge irregular fuzzy graph. Let $a_0$be the centre vertex and $a_1, a_2, \ldots, a_n$be the pendant vertices. Let $a_0 a_i = e_i, \forall 1 \le i \le n$. Suppose there exist two edges of G have same membership values. Let it be e_i, e_j and $\mu(e_i) = \mu(e_j) = r \Rightarrow d_G(e_i) = d_G(e_j) \Rightarrow d_G(e_1) + d_G(e_2) + \cdots + d_G(e_{i-1}) + d_G(e_{i+1}) + \cdots + d_G(e_j) + \cdots. + d_G(e_n) = d_G(e_1) + d_G(e_2) + \cdots + d_G(e_j) + \ldots. + d_G(e_{j-1}) + d_G(e_{j+1}) + \cdots. + d_G(e_n) \Rightarrow s_G(e_i) = s_G(e_j)$. Thus G is not support neighbourly edge irregular fuzzy graph. Which is a contradiction. Hence no two adjacent edges have same membership values.

Conversely suppose no two adjacent edges of G have same membership values. $i.e. \mu(e_i) \ne \mu(e_j), \forall\, e_i, e_j \text{ are adjacent } \Rightarrow -\mu(e_i) \ne -\mu(e_j), \forall\, e_i, e_j \text{ are adjacent } \Rightarrow \mu(e_i) + d_G(a_0) - 2\mu(e_i) \ne \mu(e_j) + d_G(a_0) - 2\mu(e_j), \forall\, e_i, e_j \text{ are adjacent } \Rightarrow d_G(a_i) + d_G(a_0) - 2\mu(e_i) \ne d_G(a_j) + d_G(a_0) - 2\mu(e_j), \forall\, e_i, e_j \text{ are adjacent } \Rightarrow d_G(e_i) \ne d_G(e_j), \forall\, e_i, e_j \text{ are adjacent } \Rightarrow d_G(e_1) + d_G(e_2) + \cdots + d_G(e_{i-1}) + d_G(e_{i+1}) + \cdots + d_G(e_j) + \cdots. + d_G(e_n) \ne d_G(e_1) + d_G(e_2) + \cdots + d_G(e_j) + \ldots. + d_G(e_{j-1}) + d_G(e_{j+1}) + \cdots. + d_G(e_n)\ \forall\, e_i, e_j \text{ are adjacent} \Rightarrow s_G(e_i) \ne s_G(e_j)\ \forall\, e_i, e_j \text{ are adjacent}$. Hence G is support highly edge irregular fuzzy graph.

Corollary 4.3. *Let $G : (\sigma,\mu)$ be a fuzzy graph on $G^*(V,E)$, a star graph $K_{1,n}$. If G is support neighbourly edge irregular fuzzy graph iff G is support neighbourly totally edge irregular fuzzy graph.*

Proof. Let $G : (\sigma,\mu)$ be a fuzzy graph on $G^*(V,E)$, a star graph $K_{1,n}$. Let u be the central vertex and $u_1, u_2, \ldots, u_n$ be the vertices adjacent to u. Also let $uu_i = e_i, \forall 1 \le i \le n$. If G is support neighbourly edge irregular fuzzy graph, then by theorem 4.2, no two adjacent edges have same membership values.

Suppose G is not support neighbourly totally edge irregular fuzzy graph. Then there exists two edges e_i, e_j such that $ts_G(e_i) = ts_G(e_j) \implies s_G(e_i) + \mu(e_i = s_G(e_j) + \mu(e_j) \implies d_G(e_1) + d_G(e_2) + \ldots + d_G(e_{i-1}) + d_G(e_{i+1}) + \ldots + d_G(e_n) + \mu(e_i = d_G(e_1) + d_G(e_2) + \ldots + d_G(e_{j-1}) + d_G(e_{j+1}) + \ldots + d_G(e_n) + \mu(e_j) \implies d_G(e_j) + \mu(e_i) = d_G(e_i) + \mu(e_j) \implies d_G(u) + d_G(u_j - 2\mu(uu_j) + \mu(e_i) = d_G(u) + d_G(u_i) - 2\mu(uu_i) + \mu(e_j) \implies \mu(uu_j) - 2\mu(uu_j) + \mu(e_i) = \mu(uu_i) - 2\mu(uu_i) + \mu(e_j) \implies -\mu(uu_j) + \mu(e_i) = -\mu(uu_i) + \mu(e_j) \implies -\mu(e_j) + \mu(e_i) = -\mu(e_i) + \mu(e_j) \implies 2\mu(e_i) = 2\mu(e_j) \implies \mu(e_i) = \mu(e_j)$. Which is a contradiction. Hence G is support neighbourly totally edge irregular fuzzy graph.

Conversely, suppose G is support neighbourly edge totally irregular fuzzy graph. Suppose $s_G(e_i) = s_G(e_j)$ for some adjacent edges $e_i, e_j \implies d_G(e_1) + d_G(e_2) + \ldots + d_G(e_{i-1}) + d_G(e_{i+1}) + \ldots + d_G(e_n) = d_G(e_1) + d_G(e_2) + \ldots + d_G(e_{j-1}) + d_G(e_{j+1}) + \ldots + d_G(e_n) \implies d_G(e_j) = d_G(e_i) \implies \mu(uu_j) = \mu(uu_i)$. Thus $d_G(e_i) + \mu(e_i) = d_G(e_j) + \mu(e_j)$ for some adjacent edges e_i, e_j. Which is a contradiction. Hence G is support neighbourly edge irregular fuzzy graph. □

Theorem 4.4. *Let $G : (\sigma,\mu)$ be a fuzzy graph on $G^*(V,E)$, a path P_n. Then G is support neighbourly edge irregular fuzzy graph if the alternate edges of G have same membership values.*

Proof. Let $G : (\sigma,\mu)$ be a fuzzy graph on $G^*(V,E)$, a path P_n. Let $a_1, a_2, \ldots, a_n$ be the vertices of G and $a_i a_{i+1} = e_i, \forall 1 \le i \le n-1$. Suppose the alternate edges of G have same membership values.

Case (i): If G is an odd path, Let $\mu(e_i) = \begin{cases} r \text{ if } i \text{ is odd} \\ 2r \text{ if } i \text{ is even} \end{cases}$.

$$\text{Then } d_G(a_i) = \begin{cases} r \text{ is } i = 1 \\ 2r \text{ if } 2 \le i \le n-1 \\ 3r \text{ if } i = n \end{cases} \text{and}$$

$$d_G(e_i) = \begin{cases} 2r \text{ is } i = 1,2,4,6,\ldots,n-3 \\ 4r \text{ if } i = 3,5,7,\ldots,n-2 \\ r \text{ if } i = n-1 \end{cases}.$$

$$\text{So } s_G(e_i) = \begin{cases} 2r \text{ is } i = 1 \\ 6r \text{ if } i = 2 \\ 4r \text{ if } i = 3,5,7,\ldots,n-4,n-1 \\ 8r \text{ if } i = 4,6,8,\ldots,n-3 \\ 3r \text{ if } i = n-2 \end{cases}.$$

Here no two adjacent edges have same support. Hence G is support edge neighbourly edge irregular fuzzy graph.

Case (ii): If G is an even path.

$$\text{Let } \mu(e_i) = \begin{cases} r \text{ if } i \text{ is odd} \\ 2r \text{ if } i \text{ is even} \end{cases}.$$

$$\text{Then } d_G(a_i) = \begin{cases} r \text{ if } i = 1,n \\ 3r \text{ if } 2 \le i \le n-1 \end{cases} \text{ and}$$

$$d_G(e_i) = \begin{cases} 2r \text{ if } i = 1,n-1,2,4,6,\ldots,n-2 \\ 4r \text{ if } i = 3,5,7,\ldots,n-3 \end{cases}.$$

$$\text{So } s_G(e_i) = \begin{cases} 2r \text{ is } i = 1,n-1 \\ 6r \text{ if } i = 2,n-2 \\ 4r \text{ if } i = 3,5,7,\ldots,n-3 \\ 8r \text{ if } i = 4,6,8,\ldots,n-4 \end{cases}$$

Here every pair of adjacent edges have distinct supports. Hence $G$is support edge neighbourly edge irregular fuzzy graph.

Theorem 4.5. *Let $G(\sigma,\mu)$be a fuzzy graph on $G^*(V,E)$, a barbell graph $B(m,n)$. Then G is support neighbourly edge irregular fuzzy graph iff no two adjacent pendant edges have same membership value and $\mu(uu_i) \neq (n-2)dG(v) + 3\mu(uv)$.*

Proof. Let $G(\sigma,\mu)$ be a fuzzy graph on $G^*(V,E)$, a barbell graph $B(m,n)$. Let u,v be the central vertices, $u_1,u_2,....,u_m$ and $v_1,v_2,...,v_n$ be the vertices adjacent to u and v respectively. Also let $uu_i = e_i, \forall 1 \leq i \leq m, uv = e_{m+1}$ and $vv_j = e_{j+m+1}, \forall 1 \leq j \leq n$. Suppose G is support neighbourly edge irregular fuzzy graph. Then $s_G(e_i) \neq s_G(e_j), \forall e_i, e_j$ are adjacent. In particular, let us take $s_G(e_i) \neq s_G(e_j), \forall 1 \leq i,j \leq m \implies d_G(e_1) + d_G(e_2) + ... + d_G(e_{i-1}) + d_G(e_{i+1}) + ... + d_G(e_j) + ... + d_G(e_m) + d_G(e_{m+1}) \neq d_G(e_1) + d_G(e_2) + ... + d_G(e_i) + ... + d_G(e_{j-1}) + d_G(e_{j+1}) + ... + d_G(e_m) + d_G(e_{m+1}), \forall 1 \leq i,j \leq m \implies d_G(e_j) \neq d_G(e_i), \forall 1 \leq i,j \leq m \implies \mu(uu_j) \neq \mu(uu_i), \forall 1 \leq i,j \leq m$. Similarly if we take $s_G(e_i) \neq s_G(e_j), \forall m+2 \leq i,j \leq m+n+1$, we also get $\mu(uu_j) \neq \mu(uu_i), \forall m+2 \leq i,j \leq m+n+1$. If we take $s_G(e_i) \neq s_G(e_j)$, where e_i is pendant edge and e_j is middle edge. Then $s_G(e_i) \neq s_G(e_j) \implies d_G(e_1) + d_G(e_2) + ... + d_G(e_{i-1}) + d_G(e_{i+1}) + ... + d_G(e_m) + d_G(e_{m+1}) \neq d_G(e_1) + d_G(e_2) + ... + d_G(e_m) + d_G(e_{m+2}) + d_G(e_{m+3}) + ... + d_G(e_{m+n+1}) \implies d_G(e_{m+1}) \neq d_G(e_i) + d_G(e_{m+2}) + d_G(e_{m+3}) + ... + d_G(e_{m+n+1}) \implies d_G(u) + d_G(v) - 2\mu(uv) \neq d_G(u) + d_G(u_i) - 2\mu(uu_i) + d_G(v) + d_G(v_1) - 2\mu(vv_1) + d_G(v) + d_G(v_2) - 2\mu(vv_2) + ... + d_G(v) + d_G(v_n) - 2\mu(vv_n) \implies -2\mu(uv) \neq d_G(u_i) - 2\mu(uu_i) + (n-1)d_G(v) + d_G(v_1) - 2\mu(vv_1) + d_G(v_2) - 2\mu(vv_2) + ... + d_G(v_n) - 2\mu(vv_n) \implies -2\mu(uv) \neq \mu(uu_i) - 2\mu(uu_i) + (n-1)d_G(v) + \mu(vv_1) - 2\mu(vv_1) + \mu(vv_2) - 2\mu(vv_2) + ... + \mu(vv_n) - 2\mu(vv_n) + 2\mu(uv) \implies -2\mu(uv) \neq -\mu(uu_i) + (n-1)d_G(v) - \mu(vv_1) - \mu(vv_2) - ..... - \mu(vv_n) \implies -\mu(uu_i) + (n-1)d_G(v) - \mu(vv_1) - \mu(vv_2) - ..... - \mu(vv_n) - \mu(uv) + \mu(uv) + 2\mu(uv) \neq 0 \implies -\mu(uu_i) + (n-1)d_G(v) - d_G(v) + 3\mu(uv) \neq 0 \implies -\mu(uu_i) + (n-2)d_G(v) + 3\mu(uv) \neq 0 \implies \mu(uu_i) \neq (n-2)d_G(v) + 3\mu(uv)$.

Theorem 4.6. *Let $G(\sigma,\mu)$ be a fuzzy graph on $G^*(V,E)$, P_n where $n = mk$. If every pair of m consecutive edges have distinct membership values and every m^{th} edge has same membership values, then G is support neighbourly edge irregular fuzzy graph.*

Proof. Let $G(\sigma,\mu)$ be a fuzzy graph on $G^*(V,E)$, P_n where $n = mk$. Let $a_1, a_2, \dots, a_n$ be the vertices of G and $a_i a_{i+1} = e_i, \forall 1 \le i \le n-1$. Suppose every pair of m consecutive edges have distinct membership values and every m^{th} edge has same membership values. Then

$$i.e\,\mu(e_i) = \begin{cases} r \text{ if } i = 1, m+1, \dots, (k-1)m+1 \\ 2r \text{ if } i = 2, m+2, \dots, (k-1)m+2 \\ 3r \text{ if } i = 3, m+3, \dots, (k-1)m+3 \\ . \\ . \\ . \\ (m-1)r \text{ if } i = m-1, 2m-1, \dots, km-1 \\ mr \text{ if } i = m, 2m, \dots, (k-1)m. \end{cases}$$

Conversely, suppose no two adjacent edges have same membership values and $\mu(uu_i) \neq (n-2)d_G(v) + 3\mu(uv)$. Without loss of generality, we may assume that $\mu(uu_i) = ir, \forall 1 \le i \le m$, $\mu(vv_j) = jr, \forall 1 \le j \le n$ and $\mu(uv) = r$. Then $d_G(u_i) = \mu(uu_i) = ir, \forall 1 \le i \le m$. Similarly $d_G(v_j) = jr, \forall 1 \le j \le n$. Consider $d_G(u) = \mu(uu_1) + \mu(uu_2) + \dots + \mu(uu_m) + \mu(uv) = r + 2r + 3r + \dots + mr + r = \frac{m(m+1)r}{2} + r$. Similarly, $d_G(v) = \frac{n(n+1)r}{2} + r$. Consider the degree of the edge $d_G(uu_i) = d_G(u) = d_G(u_i) - 2\mu(uu_i) = \frac{m(m+1)r}{2} + r + ir - 2ir = \frac{m(m+1)r}{2} + r - ir, \forall 1 \le i \le m$. Similarly $d_G(vv_j) = \frac{n(n+1)r}{2} + r - jr, \forall 1 \le j \le n$ and $d_G(uv) = \frac{m(m+1)r}{2} + \frac{n(n+1)r}{2}$. Consider the support of an edge $s_G(uu_i) = d_G(uu_1) + d_G(uu_2) + \dots + d_G(uu_{i-1}) + d_G(uu_{i+1}) + \dots + d_G(uu_m) + d_G(uv) = \frac{m(m+1)r}{2} + r - r + \frac{m(m+1)r}{2} + r - 2r + \dots + \frac{m(m+1)r}{2} + r - (i-1)r + \frac{m(m+1)r}{2} + r - (i+1)r + \dots + \frac{m(m+1)r}{2} + r - mr + \frac{m(m+1)r}{2} + \frac{n(n+1)r}{2} = \frac{(m-1)m(m+1)r}{2} + (m-1)r + ir - (r + 2r + \dots + mr) + \frac{m(m+1)r}{2} + \frac{n(n+1)r}{2} = \frac{(m-1)m(m+1)r}{2} + (m-1)r + ir - \frac{m(m+1)r}{2} + \frac{m(m+1)r}{2} + \frac{n(n+1)r}{2} = \frac{(m-1)m(m+1)r}{2} + (m-1)r + ir + \frac{n(n+1)r}{2}, \forall 1 \le i \le m$. Similarly $s_G(vv_j) = \frac{(n-1)n(n+1)r}{2} + (n-1)r + jr + \frac{m(m+1)r}{2}, \forall 1 \le j \le n$ and $s_G(uv) = \sum_{i=1}^{m} d_G(uu_i) + \sum_{j=1}^{n} d_G(vv_j) = \frac{m(m+1)r}{2} + r - r + \frac{m(m+1)r}{2} + r - 2r + \dots + \frac{m(m+1)r}{2} + r - ir + \dots + \frac{m(m+1)r}{2} + r - mr + \frac{n(n+1)r}{2} + r - r + \frac{n(n+1)r}{2} + r - 2r + \dots + \frac{n(n+1)r}{2} + r - nr = \frac{m^2(m+1)r}{2} + mr - (r + 2r + \dots + mr) + \frac{n^2(n+1)r}{2} + nr - (r + 2r + \dots + nr) = \frac{(m-1)m(m+1)r}{2} + mr + \frac{(n-1)n(n+1)r}{2} + nr$. By examining every pair of adjacent edges, we have no two adjacent edges have same support. Hence G is not support neighbourly edge irregular fuzzy graph. □

$$\text{So } d_G(a_i) = \begin{cases} r \text{ if } i = 1 \\ 3r \text{ if } i = 2, m+2, ..., (k-1)m+2 \\ 5r \text{ if } i = 3, m+3, ..., (k-1)m+3 \\ . \\ . \\ . \\ (j-1)r + jr \text{ if } i = j, m+j, ..., (k-1)m+j \\ . \\ . \\ (m-2)r + (m-1)r \text{ if } i = m-1, 2m-1, ..., km-1 \\ mr \text{ if } i = m, 2m, ..., (k-1)m \\ (m+1)r \text{ if } i = m+1, 2m+1, ..., (k-1)m+1 \\ (m-1)r \text{ if } i = km. \end{cases}$$

$$\text{and } d_G(e_i) = \begin{cases} 2r \text{ if } i = 1 \\ 4r \text{ if } i = 2, m+2, ..., (k-1)m+2 \\ 6r \text{ if } i = 3, m+3, ..., (k-1)m+3 \\ . \\ . \\ . \\ 2jr \text{ if } i = j, m+j, ..., (k-1)m+j \\ . \\ . \\ 2(m-1)r \text{ if } i = m-1, 2m-1, ..., (k-1)m-1 \\ r \text{ if } i = m, 2m, ..., (k-1)m \\ 2mr \text{ if } i = m+1, 2m+1, ..., (k-1)m+1 \\ (m-2)r \text{ if } i = km-1. \end{cases}$$

$$\text{Also } s_G(e_i) = \begin{cases} 4r \text{ if } i = 1 \\ 8r \text{ if } i = 2, m+2, ..., (k-1)m+2 \\ 12r \text{ if } i = 3, m+3, ..., (k-1)m+3 \\ . \\ . \\ . \\ 4jr \text{ if } i = j, m+j, ..., (k-1)m+j \\ . \\ . \\ 4(m-1)r \text{ if } i = m-1, 2m-1, ..., (k-1)m-1 \\ (4m-2)r \text{ if } i = m, 2m, ..., (k-1)m \\ 5r \text{ if } i = m+1, 2m+1, ..., (k-1)m+1 \\ 2(m-2)r \text{ if } i = km-1 \end{cases}$$

Here no two adjacent edges have same support. Thus G is support neighbourly edge irregular fuzzy graph.

Theorem 4.7. *Let $G(\sigma,\mu)$ be a fuzzy graph on $G^*(V,E), C_n$ where $n = mk$. If every pair of m consecutive edges have distinct membership values and every m^{th} edge has same membership values, then G is support neighbourly edge irregular fuzzygraph.*

Proof. Let $G(\sigma,\mu)$ be a fuzzy graph on $G^*(V,E), C_n$ where $n = mk$. Let $a_1, a_2, ..., a_n$ be the vertices of G and $a_i a_{i+1} = e_i, \forall 1 \leq i \leq n-1$ and $a_n a_1 = e_n$. Suppose every pair of m consecutive edges have distinct membership values and every m^{th} edge has same membership values.

$$i.e\,\mu(e_i) = \begin{cases} r \text{ if } i = 1, m+1, ..., (k-1)m+1 \\ 2r \text{ if } i = 2, m+2, ..., (k-1)m+2 \\ 3r \text{ if } i = 3, m+3, ..., (k-1)m+3 \\ . \\ . \\ . \\ (m-2)r \text{ if } i = m-2, 2m-2, ..., km-2 \\ (m-1)r \text{ if } i = m-1, 2m-1, ..., km-1 \\ mr \text{ if } i = m, 2m, ..., km. \end{cases}$$

$$\text{So } d_G(a_i) = \begin{cases} (m+1)r \text{ if } i = 1, m+1, ..., (k-1)m+1 \\ 3r \text{ if } i = 2, m+2, ..., (k-1)m+2 \\ 5r \text{ if } i = 3, m+3, ..., (k-1)m+3 \\ . \\ . \\ . \\ (j-1)r + jr \text{ if } i = j, m+j, ..., (k-1)m+j \\ . \\ . \\ (m-2)r + (m-1)r \text{ if } i = m-1, 2m-1, ..., km-1 \\ (m-1)r + mr \text{ if } i = m, 2m, ..., km. \end{cases}$$

$$\text{and } d_G(e_i) = \begin{cases} 2(m+2)r \text{ if } i = 1, m+1, ..., (k-1)m+1 \\ 4r \text{ if } i = 2, m+2, ..., (k-1)m+2 \\ 6r \text{ if } i = 3, m+3, ..., (k-1)m+3 \\ . \\ . \\ . \\ 2jr \text{ if } i = j, m+j, ..., (k-1)m+j \\ . \\ . \\ 2(m-1)r \text{ if } i = m-1, 2m-1, ..., km-1 \\ mr \text{ if } i = m, 2m, ..., km. \end{cases}$$

$$\text{Also } s_G(e_i) = \begin{cases} (m+4)r \text{ if } i = 1, m+1, ..., (k-1)m+1 \\ (m+8)r \text{ if } i = 2, m+2, ..., (k-1)m+2 \\ 12r \text{ if } i = 3, m+3, ..., (k-1)m+3 \\ . \\ . \\ . \\ 4jr \text{ if } i = j, m+j, ..., (k-1)m+j \\ . \\ . \\ 3mr - 4r \text{ if } i = m-1, 2m-1, ..., km-1 \\ 3mr \text{ if } i = m, 2m, ..., km. \end{cases}$$

Here every pair of adjacent edges have distinct supports. Thus G is support neighbourly edge irregular fuzzy graph. □

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