Original Article

Analyzation Of Extended Tri-Cum Biseries Queuing Model In A Fuzzy Environment

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Abstract

Queuing theory is the study and modelling of people who wait in lines. Fuzziness gravels a puzzling piece in relation to hazy info to produce uncertainty. A variety of varied daily situations, including media transmission systems, computer networks, manufacturing companies, traffic control, organizations and other fields and halls of fame, use queuing systems. In this extended queuing paradigm, three servers are linked in parallel in a tri cum biseries approach, which is analogous to a common place server in series. Arrivals are made in predefined batch sizes, according to the poison technique. Services are provided on a first-come, first-served basis. A generating function was used to calculate the steady state solutions and queue characteristics. There are numerous ways that can be used to address the issue of fuzzy numbers. We recommend utilizing the Wingspans ranking approach to identify the performance metrics of the provided queuing model. This wingspans ranking method converts arrival and service rate data into accurate numbers. The models' utility is proven using numerical examples.

Key Words Tri-cum Biserial queues, Moment generating function, Pentagonal fuzzy number, Intuitionistic pentagonal fuzzy number, Wingspans fuzzy ranking, Queue characteristics.

1. Introduction

A fuzzy queuing system can be thought of as a perception of a classic queuing system, which will be referred to as the original of the fuzzy queue. The models that incorporate large quantities of input or output offer a powerful foundation for analyzing system performance measures and are crucial in a variety of situations. There has been a lot of study done on the parallel and biserial waiting line models. Compared to crisp queue models, fuzzy queue models have greater absoluteness. These are really absolute in the conditions that exist in the real world. Fuzzy sets were created by L.A. Zadeh as an extension of the classical set in 1965. The fuzzy queue models were first introduced to Lie and Lee in 1989. Following that a number of researchers then focused their efforts on fuzzy queues.

Atanassov created intuitionistic fuzzy sets (IFS), a generalization of fuzzy sets that incorporates the degree of non-membership function. Intriguing and helpful in a wide range of application domains is the idea of defining intuitionistic fuzzy sets. The intuitionistic fuzzy set is used to describe practical issues in fields including marketing, psychology, financial services, medical diagnostics, sales analysis and so forth. Wang and Westman introduced a new notion, the w-center, for ordering triangular and trapezoidal fuzzy numbers. Based on the concept of the w-center, we created a new ranking for intuitionistic pentagonal fuzzy numbers. The fuzzy number is ranked precisely and distinctly**.** A parallel queue network model with static batch arrival was created by Gupta et al. In the current study, the tri-cum biseries queue model is expanded. Our paper combined a tri-cum biseries queuing network in a fuzzy environment. This model can be applied in a variety of settings that are similar, including sports clubs, shopping precincts, banking sectors, computer networks, and many more.

2. Preliminaries

2.1. Intuitionistic fuzzy set

Let $\tilde{C}^I$ be an intuitionistic fuzzy set in universe of discourse U and defined by a set of ordered triple $\tilde{C}^I = \left\{\left(x, \mu_{\tilde{C}^I}(x), N_{\tilde{C}^I}(x)\right); x \in U\right\}$ where the functions $\mu_{\tilde{C}^I}(x), N_{\tilde{C}^I}(x)$ is a mapping from $U \to [0,1]$, such that $0 \le \mu_{\tilde{C}^I}(x) + N_{\tilde{C}^I}(x) \le 1, \forall\, x \in U.$ The membership degree of the element $x \in U$ in $\tilde{C}^I$ represents as $\mu_{\tilde{C}^I}(x).$ The non-membership degree of the element $x \in U$ in $\tilde{C}^I$ represents as $N_{\tilde{C}^I}(x).$ The hesitation degree of $\tilde{C}^I$ is $\psi(x) = 1 - \mu_{\tilde{C}^I}(x) - N_{\tilde{C}^I}(x), \forall\, x \in U.$

2.2. Intuitionistic fuzzy number

An intuitionistic fuzzy set $\tilde{C}^I = \left\{\left(x, \mu_{\tilde{C}^I}(x), N_{\tilde{C}^I}(x)\right); x \in X\right\}$ is said to an Intuitionistic fuzzy number, if the following conditions are hold:

(i) The mean value of $\tilde{C}^I$ is f, which is normal and belongs to R such that $\mu_{\tilde{C}^I}(f) = 1$ and $N_{\tilde{C}^I}(f) = 0$

(ii) $\mu_{\tilde{C}^I}(f)$ and $N_{\tilde{C}^I}(f)$ are piecewise continuous functions from $R \to [0,1]$ and $0 \le \mu_{\tilde{C}^I}(x) + N_{\tilde{C}^I}(x) \le 1, \forall\, x \in R,$ with

$$\mu_{\tilde{C}^I}(x) = \begin{cases} d_1(x), & f - c_a \le x \le f \\ 1, & x = f \\ e_1(x), & f < x \le f + c_b \\ 0, & otherwise \end{cases}$$

Where the functions $d_1(x)$ and $e_1(x)$ are piece-wise continuous, strictly increasing and strictly decreasing function in $[f - c_a, f)$ and $(f, f + c_b]$

$$N_{\tilde{C}^I}(x) = \begin{cases} d_2(x), & f - c'_a \le x \le f;\ 0 \le d_1(x) + d_2(x) \le 1 \\ 0, & x = f \\ e_2(x), & f < x \le f + c'_b;\ 0 \le e_1(x) + e_2(x) \le 1 \\ 1, & otherwise \end{cases}$$

Where $d_2(x)$ and $e_2(x)$ are piece-wise continuous, strictly decreasing and strictly increasing function $[f - c'_a, f]$ and $[f, f + c_b']$ respectively. The Intuitionistic fuzzy number $\tilde{C}^I$ is denoted by $\tilde{C}^I = (f\ ; c_a, c_b; c_a', c_b')$.

2.3. Pentagonal Intuitionistic fuzzy number

A pentagonal intuitionistic fuzzy number of an intuitionistic fuzzy set $\tilde{C}^I$ is denoted as $\tilde{C}^{PI} = (c_a, c_b, c_c, c_d, c_e; c_a', c_b', c_c', c_d', c_e')$ where $c_i, c_i' \in R, i = a, b, c, d, e$ and its membership and non-membership function are defined by

$$\mu_{\tilde{C}^{PI}}(x) = \begin{cases} 0, & x < c_a \\ \frac{1}{2}\left(\frac{x - c_a}{c_b - c_a}\right), & c_a \le x \le c_b \\ \frac{1}{2} + \frac{1}{2}\left(\frac{x - c_b}{c_c - c_b}\right), & c_b \le x \le c_c \\ 1, & x = c_c \\ \frac{1}{2} + \frac{1}{2}\left(\frac{c_d - x}{c_d - c_c}\right), & c_c \le x \le c_d \\ \frac{1}{2}\left(\frac{c_e - x}{c_e - c_d}\right), & c_d \le x \le c_e \\ 0, & x \ge c_e \end{cases}$$

and

$$N_{\tilde{C}^{PI}}(x) = \begin{cases} 1, & x < c_a' \\ \frac{1}{2} + \frac{1}{2}\left(\frac{c_b' - x}{c_b' - c_a'}\right), & c_a' \le x \le c_b' \\ \frac{1}{2}\left(\frac{c_c' - x}{c_c' - c_b'}\right), & c_b' \le x \le c_c' \\ 0, & x = c_c' \\ \frac{1}{2}\left(\frac{x - c_c'}{c_d' - c_c'}\right), & c_c' \le x \le c_d' \\ \frac{1}{2} + \frac{1}{2}\left(\frac{x - c_d'}{c_e' - c_d'}\right), & c_d' \le x \le c_e' \\ 1, & x > c_e' \end{cases}$$

3. Mathematical Modelling

Three servers are related in parallel in a tri cum biseries way in this queuing system. The
Currently taken into account network model consists of two queuing subsystems θ_1 and θ_3 each of which is centrally connected to a single server θ_2 . The three parallel servers S_1, S_2 and S_3 make up the subsystem θ_1 and the other three parallel servers S_5, S_6 and S_7 make up the subsystem θ_3. Customers entered the system with implied arrival rates λ_1, λ_2 and λ_3 arrive in bunches of fixed sizes B_a, B_b and B_c follow the Poisson process and form queues. The customers r_1 arriving at arrival rate λ_1, after receiving all services from server S_1 can use the power available at servers S_2 or S_3 (both or either of two) with the probabilities p_{12} and p_{13} or simultaneously can utilize the power available at server S_4 in θ_2 with the probability p_{14} to such a quantity that $p_{12} + p_{13} + p_{14} = 1$. Customers approaching at S_2 with mean rate λ_2 after being served there, either will shift to S_1 with the probability p_{21} or join S_3 with the probability p_{23} or may join S_4 with the probability p_{24} such that $p_{21} + p_{23} + p_{24} = 1$.

Similar approach are used for S_3 with arrival rate λ_3, so that $p_{31} + p_{32} + p_{34} = 1$. After receiving service at S_4, customers will likely join either S_5 or S_6 or S_7 with a chance that $p_{45} + p_{46} + p_{47} = 1$. In S_5, the consumer has the option of moving to S_6 with probability p_{56} or S_7 with probability p_{57} or leaving the system with probability p_{55} such that $p_{55} + p_{56} + p_{57} = 1$. A comparable criterion will follow to the servers S_6 and S_7 such that $p_{65} + p_{66} + p_{67} = 1$ & $p_{75} + p_{76} + p_{77} = 1$.

The assumed average service rates at the service channels $S_1, S_2, S_3, S_4, S_5, S_6$ *and* S_7 are presummated as $\mu_1, \mu_2, \mu_3, \mu_4, \mu_5, \mu_6$ *and* μ_7 respectively. The model formulation is clearly illustrated in Figure 1 together with all of the transitional stages, associated probabilities, and terminology.

4. Steady-state Analysis

The probability that there are $r_1, r_2, r_3, r_4, r_5, r_6$ *and* r_7 customers at any time t waiting in queues is equal to $P_{r_1,r_2,r_3,r_4,r_5,r_6,r_7}(t)$, where $r_1 > B_a, r_2 > B_b, r_3 > B_c$ *and* $r_4, r_5, r_6, r_7 > 0$.

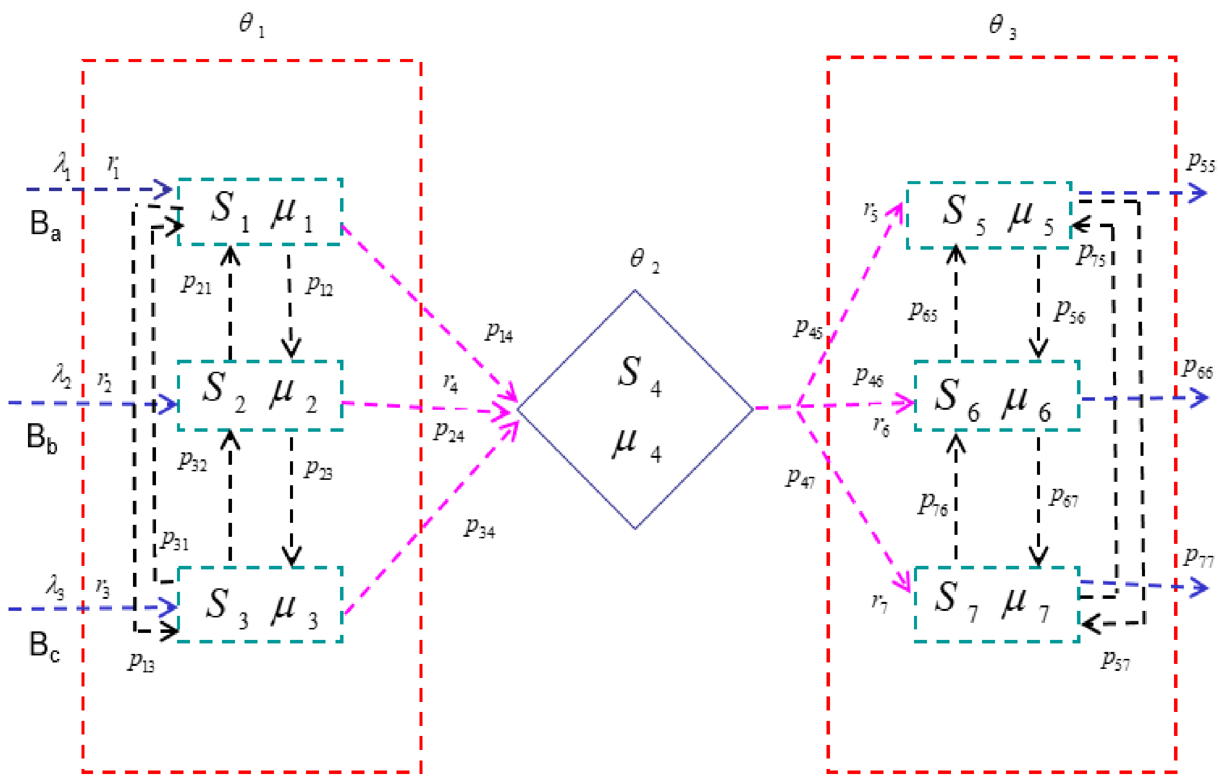

Figure 1: Tri-Cum Biserial Queuing Network

$$
\begin{aligned}
&\left(\lambda_1+\lambda_2+\lambda_3\right)+(\mu_1+\mu_2+\mu_3+\mu_4+\mu_5+\mu_6+\mu_7)P_{r_1,r_2,r_3,r_4,r_5,r_6,r_7} = \\
&\quad \lambda_1 P_{r_1-B_a,r_2,r_3,r_4,r_5,r_6,r_7} + \lambda_2 P_{r_1,r_2-B_b,r_3,r_4,r_5,r_6,r_7} + \lambda_3 P_{r_1,r_2,r_3-B_c,r_4,r_5,r_6,r_7} + \mu_1 p_{12} P_{r_1+1,r_2-1,r_3,r_4,r_5,r_6,r_7} \\
&\quad + \mu_1 p_{13} P_{r_1+1,r_2,r_3-1,r_4,r_5,r_6,r_7} + \mu_1 p_{14} P_{r_1+1,r_2,r_3,r_4-1,r_5,r_6,r_7} + \mu_2 p_{21} P_{r_1-1,r_2+1,r_3,r_4,r_5,r_6,r_7} + \mu_2 p_{23} P_{r_1,r_2+1,r_3-1,r_4,r_5,r_6,r_7} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1) \\
&\quad + \mu_2 p_{24} P_{r_1,r_2+1,r_3,r_4-1,r_5,r_6,r_7} + \mu_3 p_{31} P_{r_1-1,r_2,r_3+1,r_4,r_5,r_6,r_7} + \mu_3 p_{32} P_{r_1,r_2-1,r_3+1,r_4,r_5,r_6,r_7} + \mu_3 p_{34} P_{r_1,r_2,r_3+1,r_4-1,r_5,r_6,r_7} \\
&\quad + \mu_4 p_{45} P_{r_1,r_2,r_3,r_4+1,r_5-1,r_6,r_7} + \mu_4 p_{46} P_{r_1,r_2,r_3,r_4+1,r_5,r_6-1,r_7} + \mu_4 p_{47} P_{r_1,r_2,r_3,r_4+1,r_5,r_6,r_7-1} + \mu_5 p_{56} P_{r_1,r_2,r_3,r_4,r_5+1,r_6-1,r_7} \\
&\quad + \mu_5 p_{57} P_{r_1,r_2,r_3,r_4,r_5+1,r_6,r_7-1} + \mu_5 p_{55} P_{r_1,r_2,r_3,r_4,r_5+1,r_6,r_7} + \mu_6 p_{65} P_{r_1,r_2,r_3,r_4,r_5-1,r_6+1,r_7} + \mu_6 p_{66} P_{r_1,r_2,r_3,r_4,r_5,r_6+1,r_7} \\
&\quad + \mu_6 p_{67} P_{r_1,r_2,r_3,r_4,r_5,r_6+1,r_7-1} + \mu_7 p_{75} P_{r_1,r_2,r_3,r_4,r_5-1,r_6,r_7+1} + \mu_7 p_{76} P_{r_1,r_2,r_3,r_4,r_5,r_6-1,r_7+1} + \mu_7 p_{77} P_{r_1,r_2,r_3,r_4,r_5,r_6,r_7+1}
\end{aligned}
$$

Using the generating function technique, the difference equations that were obtained in steady-state form were resolved. It is defined as

$$
H\left(X_1,X_2,X_3,X_4,X_5,X_6,X_7\right) = \sum_{r_1=0}^{\infty}\sum_{r_2=0}^{\infty}\sum_{r_3=0}^{\infty}\sum_{r_4=0}^{\infty}\sum_{r_5=0}^{\infty}\sum_{r_6=0}^{\infty}\sum_{r_7=0}^{\infty} P_{r_1,r_2,r_3,r_4,r_5,r_6,r_7} \times X_1^{r_1} X_2^{r_2} X_3^{r_3} X_4^{r_5} X_5^{r_6} X_6^{r_7} X_7^{r_7} \quad \ldots\ldots\ldots\ldots (2)
$$

Such that, $|X_1| = |X_2| = |X_3| = |X_4| = |X_5| = |X_6| = |X_7| \le 1$

Partial generating functions are further defined as follows

$$H_{r_2,r_3,r_4,r_5,r_6,r_7}(X_1)=\sum_{r_1=0}^{\infty}P_{r_1,r_2,r_3,r_4,r_5,r_6,r_7}\,.\,X_1^{r_1}$$

$$H_{r_3,r_4,r_5,r_6,r_7}(X_1,X_2)=\sum_{r_2=0}^{\infty}P_{r_2,r_3,r_4,r_5,r_6,r_7}\,.\,(X_1)X_2^{r_2}$$

$$H_{r_4,r_5,r_6,r_7}(X_1,X_2,X_3)=\sum_{r_3=0}^{\infty}P_{r_3,r_4,r_5,r_6,r_7}\,.\,(X_1X_2)X_3^{r_3}$$

$$H_{r_5,r_6,r_7}(X_1,X_2,X_3,X_4)=\sum_{r_4=0}^{\infty}P_{r_4,r_5,r_6,r_7}\,.\,(X_1X_2X_3)X_4^{r_4} \qquad \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

$$H_{r_6,r_7}(X_1,X_2,X_3,X_4,X_5)=\sum_{r_5=0}^{\infty}P_{r_5,r_6,r_7}\,.\,(X_1X_2X_3X_4)X_5^{r_5}$$

$$H_{r_7}(X_1,X_2,X_3,X_4,X_5,X_6)=\sum_{r_6=0}^{\infty}P_{r_6,r_7}\,.\,(X_1X_2X_3X_4X_5)X_6^{r_6}$$

$$H(X_1,X_2,X_3,X_4,X_5,X_6,X_7)=\sum_{r_7=0}^{\infty}P_{r_7}\,.\,(X_1X_2X_3X_4X_5X_6)X_7^{r_7}$$

Solving the difference equations with the help of equation (2) & (3); we obtained equation is as follows

$$\big((\lambda_1+\lambda_2+\lambda_3)+(\mu_1+\mu_2+\mu_3+\mu_4+\mu_5+\mu_6+\mu_7)\big)H(X_1,X_2,X_3,X_4,X_5,X_6,X_7)-\mu_1H_1-\mu_2H_2-\mu_3H_3-\mu_4H_4-\mu_5H_5-\mu_6H_6-\mu_7H_7$$
$$=\lambda_1X_1^{B_a}H(X_1,X_2,X_3,X_4,X_5,X_6,X_7)+\lambda_2X_2^{B_b}H(X_1,X_2,X_3,X_4,X_5,X_6,X_7)+\lambda_3X_3^{B_c}H(X_1,X_2,X_3,X_4,X_5,X_6,X_7)$$
$$+\frac{\mu_1p_{12}}{X_1}X_2\left[H(X_1,X_2,X_3,X_4,X_5,X_6,X_7)-H(X_2,X_3,X_4,X_5,X_6,X_7)\right]+\frac{\mu_1p_{13}}{X_1}X_3\left[H(X_1,X_2,X_3,X_4,X_5,X_6,X_7)-H(X_2,X_3,X_4,X_5,X_6,X_7)\right]$$
$$+\frac{\mu_1p_{14}}{X_1}X_4\left[H(X_1,X_2,X_3,X_4,X_5,X_6,X_7)-H(X_2,X_3,X_4,X_5,X_6,X_7)\right]+\frac{\mu_2p_{21}}{X_2}X_1\left[H(X_1,X_2,X_3,X_4,X_5,X_6,X_7)-H(X_1,X_3,X_4,X_5,X_6,X_7)\right]$$
$$+\frac{\mu_2p_{23}}{X_2}X_3\left[H(X_1,X_2,X_3,X_4,X_5,X_6,X_7)-H(X_1,X_3,X_4,X_5,X_6,X_7)\right]+\frac{\mu_2p_{24}}{X_2}X_4\left[H(X_1,X_2,X_3,X_4,X_5,X_6,X_7)-H(X_1,X_3,X_4,X_5,X_6,X_7)\right]$$
$$+\frac{\mu_3p_{31}}{X_3}X_1\left[H(X_1,X_2,X_3,X_4,X_5,X_6,X_7)-H(X_1,X_2,X_4,X_5,X_6,X_7)\right]+\frac{\mu_3p_{32}}{X_3}X_2\left[H(X_1,X_2,X_3,X_4,X_5,X_6,X_7)-H(X_1,X_2,X_4,X_5,X_6,X_7)\right]$$
$$+\frac{\mu_3p_{34}}{X_3}X_4\left[H(X_1,X_2,X_3,X_4,X_5,X_6,X_7)-H(X_1,X_2,X_4,X_5,X_6,X_7)\right]+\frac{\mu_4p_{46}}{X_4}X_6\left[H(X_1,X_2,X_3,X_4,X_5,X_6,X_7)-H(X_1,X_2,X_3,X_5,X_6,X_7)\right]$$
$$+\frac{\mu_4p_{47}}{X_4}X_7\left[H(X_1,X_2,X_3,X_4,X_5,X_6,X_7)-H(X_1,X_2,X_3,X_5,X_6,X_7)\right]+\frac{\mu_4p_{45}}{X_4}X_5\left[H(X_1,X_2,X_3,X_4,X_5,X_6,X_7)-H(X_1,X_2,X_3,X_5,X_6,X_7)\right]$$
$$+\frac{\mu_5p_{56}}{X_5}X_6\left[H(X_1,X_2,X_3,X_4,X_5,X_6,X_7)-H(X_1,X_2,X_3,X_4,X_6,X_7)\right]+\frac{\mu_5p_{57}}{X_5}X_7\left[H(X_1,X_2,X_3,X_4,X_5,X_6,X_7)-H(X_1,X_2,X_3,X_4,X_6,X_7)\right]$$
$$+\frac{\mu_5p_{55}}{X_5}\left[H(X_1,X_2,X_3,X_4,X_5,X_6,X_7)-H(X_1,X_2,X_3,X_4,X_6,X_7)\right]+\frac{\mu_6p_{65}}{X_6}X_5\left[H(X_1,X_2,X_3,X_4,X_5,X_6,X_7)-H(X_1,X_2,X_3,X_4,X_5,X_7)\right]$$
$$+\frac{\mu_6p_{66}}{X_6}\left[H(X_1,X_2,X_3,X_4,X_5,X_6,X_7)-H(X_1,X_2,X_3,X_4,X_5,X_7)\right]+\frac{\mu_6p_{67}}{X_6}X_7\left[H(X_1,X_2,X_3,X_4,X_5,X_6,X_7)-H(X_1,X_2,X_3,X_4,X_5,X_7)\right]$$
$$+\frac{\mu_7p_{75}}{X_7}X_5\left[H(X_1,X_2,X_3,X_4,X_5,X_6,X_7)-H(X_1,X_2,X_3,X_4,X_5,X_6)\right]+\frac{\mu_7p_{76}}{X_7}X_5\left[H(X_1,X_2,X_3,X_4,X_5,X_6,X_7)-H(X_1,X_2,X_3,X_4,X_5,X_6)\right]$$
$$+\frac{\mu_7p_{77}}{X_7}\left[H(X_1,X_2,X_3,X_4,X_5,X_6,X_7)-H(X_1,X_2,X_3,X_4,X_5,X_6)\right]$$

Consider

$$H(X_2,X_3,X_4,X_5,X_6,X_7)=H_1;$$
$$H(X_1,X_3,X_4,X_5,X_6,X_7)=H_2;$$
$$H(X_1,X_2,X_4,X_5,X_6,X_7)=H_3;$$
$$H(X_1,X_2,X_3,X_5,X_6,X_7)=H_4;$$
$$H(X_1,X_2,X_3,X_4,X_6,X_7)=H_5;$$
$$H(X_1,X_2,X_3,X_4,X_5,X_7)=H_6;$$
$$H(X_1,X_2,X_3,X_4,X_5,X_6)=H_7$$

$$\Big((\lambda_1+\lambda_2+\lambda_3)+(\mu_1+\mu_2+\mu_3+\mu_4+\mu_5+\mu_6+\mu_7)\Big)H(X_1,X_2,X_3,X_4,X_5,X_6,X_7)-\mu_1H_1-\mu_2H_2-\mu_3H_3-$$
$$\mu_4H_4-\mu_5H_5-\mu_6H_6-\mu_7H_7$$
$$=\lambda_1X_1^{B_a}H+\lambda_2X_2^{B_b}H+\lambda_3X_3^{B_c}H+\frac{\mu_1p_{12}}{X_1}X_2\left[H-H_1\right]+\frac{\mu_1p_{13}}{X_1}X_3\left[H-H_1\right]+\frac{\mu_1p_{14}}{X_1}X_4\left[H-H_1\right]$$
$$+\frac{\mu_2p_{21}}{X_2}X_1\left[H-H_2\right]+\frac{\mu_2p_{23}}{X_2}X_3\left[H-H_2\right]+\frac{\mu_2p_{24}}{X_2}X_4\left[H-H_2\right]+\frac{\mu_3p_{31}}{X_3}X_1\left[H-H_3\right]+\frac{\mu_3p_{32}}{X_3}X_2\left[H-H_3\right]+\frac{\mu_3p_{34}}{X_3}X_4\left[H-H_3\right]$$
$$+\frac{\mu_4p_{45}}{X_4}X_5\left[H-H_4\right]+\frac{\mu_4p_{46}}{X_4}X_6\left[H-H_4\right]+\frac{\mu_4p_{47}}{X_4}X_7\left[H-H_4\right]+\frac{\mu_5p_{56}}{X_5}X_6\left[H-H_5\right]+\frac{\mu_5p_{57}}{X_5}X_7\left[H-H_5\right]+\frac{\mu_5p_{55}}{X_5}\left[H-H_5\right]$$
$$+\frac{\mu_6p_{65}}{X_6}X_5\left[H-H_6\right]+\frac{\mu_6p_{66}}{X_6}\left[H-H_6\right]+\frac{\mu_6p_{67}}{X_6}X_7\left[H-H_6\right]+\frac{\mu_7p_{75}}{X_7}X_5\left[H-H_7\right]+\frac{\mu_7p_{76}}{X_7}X_5\left[H-H_6\right]+\frac{\mu_7p_{77}}{X_7}\left[H-H_6\right]$$

$$H\Big((\lambda_1+\lambda_2+\lambda_3+\mu_1+\mu_2+\mu_3+\mu_4+\mu_5+\mu_6+\mu_7)-\lambda_1X_1^{B_a}H-\lambda_2X_2^{B_b}H-\lambda_3X_3^{B_c}H\Big)-\mu_1H_1-\mu_2H_2-\mu_3H_3-$$
$$\mu_4H_4-\mu_5H_5-\mu_6H_6-\mu_7H_7=\frac{\mu_1p_{12}}{X_1}X_2\left[H-H_1\right]+\frac{\mu_1p_{13}}{X_1}X_3\left[H-H_1\right]+\frac{\mu_1p_{14}}{X_1}X_4\left[H-H_1\right]+\frac{\mu_2p_{21}}{X_2}X_1\left[H-H_2\right]+\frac{\mu_2p_{23}}{X_2}X_3\left[H-H_2\right]$$
$$+\frac{\mu_2p_{24}}{X_2}X_4\left[H-H_2\right]+\frac{\mu_3p_{31}}{X_3}X_1\left[H-H_3\right]+\frac{\mu_3p_{32}}{X_3}X_2\left[H-H_3\right]+\frac{\mu_3p_{34}}{X_3}X_4\left[H-H_3\right]+\frac{\mu_4p_{45}}{X_4}X_5\left[H-H_4\right]$$
$$+\frac{\mu_4p_{46}}{X_4}X_6\left[H-H_4\right]+\frac{\mu_4p_{47}}{X_4}X_7\left[H-H_4\right]+\frac{\mu_5p_{56}}{X_5}X_6\left[H-H_5\right]+\frac{\mu_5p_{57}}{X_5}X_7\left[H-H_5\right]+\frac{\mu_5p_{55}}{X_5}\left[H-H_5\right]+\frac{\mu_6p_{65}}{X_6}X_5\left[H-H_6\right]$$
$$+\frac{\mu_6p_{66}}{X_6}\left[H-H_6\right]+\frac{\mu_6p_{67}}{X_6}X_7\left[H-H_6\right]+\frac{\mu_7p_{75}}{X_7}X_5\left[H-H_7\right]+\frac{\mu_7p_{76}}{X_7}X_5\left[H-H_7\right]+\frac{\mu_7p_{77}}{X_7}\left[H-H_7\right]$$

$$
H\begin{pmatrix}
\lambda_1+\lambda_2+\lambda_3+\mu_1+\mu_2+\mu_3+\mu_4+\mu_5+\mu_6+\mu_7-\lambda_1 X_1^{B_a}-\lambda_2 X_2^{B_b}-\lambda_3 X_3^{B_c}-\frac{\mu_1 p_{12}}{X_1}X_2-\frac{\mu_1 p_{13}}{X_1}X_3 \\
-\frac{\mu_1 p_{14}}{X_1}X_4-\frac{\mu_2 p_{21}}{X_2}X_1-\frac{\mu_2 p_{23}}{X_2}X_3-\frac{\mu_2 p_{24}}{X_2}X_4-\frac{\mu_3 p_{31}}{X_3}X_1-\frac{\mu_3 p_{32}}{X_3}X_2-\frac{\mu_3 p_{34}}{X_3}X_4-\frac{\mu_4 p_{45}}{X_4}X_5 \\
-\frac{\mu_4 p_{46}}{X_4}X_6-\frac{\mu_4 p_{47}}{X_4}X_7-\frac{\mu_5 p_{56}}{X_5}X_6-\frac{\mu_5 p_{57}}{X_5}X_7-\frac{\mu_5 p_{55}}{X_5}-\frac{\mu_6 p_{65}}{X_6}X_5-\frac{\mu_6 p_{66}}{X_6}-\frac{\mu_6 p_{67}}{X_6}X_7-\frac{\mu_7 p_{75}}{X_7}X_5-\frac{\mu_7 p_{76}}{X_7}X_5-\frac{\mu_7 p_{77}}{X_7}
\end{pmatrix}
$$

$$
=\mu_1 H_1\left(1-\frac{p_{12}}{X_1}X_2-\frac{\mu_1 p_{13}}{X_1}X_3-\frac{\mu_1 p_{14}}{X_1}X_4\right)+\mu_2 H_2\left(1-\frac{p_{21}}{X_2}X_1-\frac{p_{23}}{X_2}X_3-\frac{\mu_2 p_{24}}{X_2}X_4\right)+\mu_3 H_3\left(1-\frac{p_{31}}{X_3}X_1-\frac{p_{32}}{X_3}X_2-\frac{p_{34}}{X_3}X_4\right)
$$

$$
+\mu_4 H_4\left(1-\frac{p_{45}}{X_4}X_5-\frac{p_{46}}{X_4}X_6-\frac{p_{47}}{X_4}X_7\right)+\mu_5 H_5\left(1-\frac{p_{56}}{X_5}X_6-\frac{p_{57}}{X_5}X_7-\frac{p_{55}}{X_5}\right)+\mu_6 H_6\left(1-\frac{p_{65}}{X_6}X_5-\frac{\mu_6 p_{66}}{X_6}-\frac{\mu_6 p_{67}}{X_6}X_7\right)
$$

$$
+\mu_7 H_7\left(1-\frac{p_{75}}{X_7}X_5-\frac{p_{76}}{X_7}X_5-\frac{p_{77}}{X_7}\right)
$$

$$
H=\left(\frac{\begin{matrix}\mu_1 H_1\left(1-\frac{p_{12}}{X_1}-\frac{\mu_1 p_{13}}{X_1}X_3-\frac{\mu_1 p_{14}}{X_1}X_4\right)+\mu_2 H_2\left(1-\frac{p_{21}}{X_2}X_1-\frac{p_{23}}{X_2}X_3-\frac{\mu_2 p_{24}}{X_2}X_4\right)+\mu_3 H_3\left(1-\frac{p_{31}}{X_3}X_1-\frac{p_{32}}{X_3}X_2-\frac{p_{34}}{X_3}X_4\right)\\ +\mu_4 H_4\left(1-\frac{p_{45}}{X_4}X_5-\frac{p_{46}}{X_4}X_6-\frac{p_{47}}{X_4}X_7\right)+\mu_5 H_5\left(1-\frac{p_{56}}{X_5}X_6-\frac{p_{57}}{X_5}X_7-\frac{p_{55}}{X_5}\right)+\mu_6 H_6\left(1-\frac{p_{65}}{X_6}X_5-\frac{\mu_6 p_{66}}{X_6}-\frac{\mu_6 p_{67}}{X_6}X_7\right)+\mu_7 H_7\left(1-\frac{p_{75}}{X_7}X_5-\frac{p_{76}}{X_7}X_5-\frac{p_{77}}{X_7}\right)\end{matrix}}{\begin{matrix}\lambda_1(1-X_1^{B_A})+\lambda_2(1-X_2^{B_B})+\lambda_3(1-X_3^{B_C})+\mu_1\left\{1-\frac{p_{12}}{X_1}X_2-\frac{p_{13}}{X_1}X_3-\frac{p_{14}}{X_1}X_4\right\}+\mu_2\left\{1-\frac{p_{21}}{X_2}X_1-\frac{p_{23}}{X_2}X_3-\frac{p_{24}}{X_2}X_4\right\}+\mu_3\left\{1-\frac{p_{31}}{X_3}X_1-\frac{p_{32}}{X_3}X_2-\frac{p_{34}}{X_3}X_4\right\}\\ +\mu_4\left\{1-\frac{p_{45}}{X_4}X_5--\frac{p_{46}}{X_4}X_6-\frac{p_{47}}{X_4}X_7\right\}+\mu_5\left\{1-\frac{p_{56}}{X_5}X_6-\frac{p_{57}}{X_5}X_7-\frac{p_{55}}{X_5}\right\}+\mu_6\left\{1`-\frac{p_{65}}{X_6}X_5-\frac{p_{66}}{X_6}-\frac{p_{67}}{X_6}X_7\right\}+\mu_7\left\{1-\frac{p_{75}}{X_7}X_5-\frac{p_{76}}{X_7}X_6-\frac{p_{77}}{X_7}\right\}\end{matrix}}\right)
$$

If $X_1=X_2=X_3=X_4=X_5=X_6=X_7=1$ then H tends to an indeterminate form.

Taking limit $X_1\to X_2\to X_3\to X_4\to X_5\to X_6\to X_7\to 1$

$$
H\left(X_1,X_2,X_3,X_4,X_5,X_6,X_7\right)=\begin{cases}1; & r_1,r_2,r_3,r_4,r_5,r_6,r_7\neq 0\\ 0; & otherwise\end{cases}
$$

$H\left(1,1,1,1,1,1,1\right)=1$ Considering $X_1=1\ as\ X_2,X_3,X_4,X_5,X_6,X_7\to 1$

Differentiate with respect to $'X_1'$ we get,

$$
1\ =\frac{\mu_1 H_1\left(p_{12}+p_{13}+p_{14}\right)+\mu_2 H_2\left(-p_{21}\right)+\mu_3 H_3\left(-p_{31}\right)}{-\lambda_1 B_a+\mu_1\left(p_{12}+p_{13}+p_{14}\right)+\mu_2\left(-p_{21}\right)+\mu_3\left(-p_{31}\right)}
$$

$$
\mu_1 H_1-\mu_2 H_2 p_{21}-\mu_3 H_3 p_{31}=-\lambda_1 B_a+\mu_1-\mu_2 p_{21}-\mu_3 p_{31} \qquad \text{.............................(4)}
$$

Differentiate with respect to $'X_2'$ we get,

$$1 = \frac{\mu_1 H_1 \left(-p_{12}\right) + \mu_2 H_2 \left(p_{21} + p_{23} + p_{24}\right) + \mu_3 H_3 \left(-p_{32}\right)}{-\lambda_2 B_b + \mu_1 \left(-p_{12}\right) + \mu_2 \left(p_{21} + p_{23} + p_{24}\right) + \mu_3 \left(-p_{32}\right)}$$

$$-\mu_1 H_1 P_{12} + \mu_2 H_2 - \mu_3 H_3 p_{32} = -\lambda_2 B_b - \mu_1 P_{12} + \mu_2 - \mu_3 p_{32} \qquad \text{..............................(5)}$$

Differentiate with respect to $'X_3'$ we get,

$$1 = \frac{\mu_1 H_1 \left(-p_{13}\right) + \mu_2 H_2 \left(-p_{23}\right) + \mu_3 H_3 \left(p_{31} + p_{32} + p_{34}\right)}{-\lambda_3 B_c + \mu_1 \left(-p_{13}\right) + \mu_2 \left(-p_{23}\right) + \mu_3 \left(p_{31} + p_{32} + p_{34}\right)}$$

$$-\mu_1 H_1 P_{13} - \mu_2 H_2 p_{23} + \mu_3 H_3 = -\lambda_3 B_c - \mu_1 P_{13} - \mu_2 P_{23} + \mu_3 \qquad \text{..............................(6)}$$

Differentiate with respect to $'X_4'$ we get,

$$1 = \frac{\mu_3 H_3 \left(-p_{34}\right) + \mu_4 H_4 \left(p_{45} + p_{46} + p_{47}\right) + \mu_1 H_1 \left(-p_{14}\right) + \mu_2 H_2 \left(-p_{24}\right)}{\mu_1 \left(-p_{14}\right) + \mu_2 \left(-p_{24}\right) + \mu_3 \left(-p_{34}\right) + \mu_4 \left(p_{45} + p_{46} + p_{47}\right)}$$

$$-\mu_1 H_1\, p_{14} - \mu_2 H_2 p_{24} - \mu_3 H_3 p_{34} + \mu_4 H_4 = -\mu_1 P_{14} - \mu_2 p_{24} - \mu_3 p_{34} + \mu_4 \qquad \text{..............................(7)}$$

Differentiate with respect to $'X_5'$ we get,

$$1 = \frac{\mu_4 H_4 \left(-p_{45}\right) + \mu_5 H_5 \left(p_{56} + p_{57} + p_{55}\right) + \mu_6 H_6 \left(-p_{65}\right) + \mu_7 H_7 \left(-p_{75}\right)}{\mu_4 \left(-p_{45}\right) + \mu_5 \left(p_{56} + p_{57} + p_{55}\right) + \mu_6 \left(-p_{65}\right) + \mu_7 \left(-p_{75}\right)}$$

$$-\mu_4 H_4\, p_{45} + \mu_5 H_5 - \mu_6 H_6 p_{65} - \mu_7 H_7 p_{75} = -\mu_4 P_{45} - \mu_6 p_{65} - \mu_7 p_{75} + \mu_5 \qquad \text{..............................(8)}$$

Differentiate with respect to $'X_6'$ we get,

$$1 = \frac{\mu_4 H_4 \left(-p_{46}\right) + \mu_5 H_5 \left(-p_{56}\right) + \mu_6 H_6 \left(p_{65} + p_{66} + p_{67}\right) + \mu_7 H_7 \left(-p_{76}\right)}{\mu_4 \left(-p_{46}\right) + \mu_5 \left(-p_{56}\right) + \mu_6 \left(p_{65} + p_{66} + p_{67}\right) + \mu_7 \left(-p_{76}\right)}$$

$$-\mu_4 H_4\, p_{46} - \mu_5 H_5 p_{56} + \mu_6 H_6 - \mu_7 H_7 p_{76} = -\mu_4 P_{46} - \mu_5 p_{56} - \mu_7 p_{76} + \mu_6 \qquad \text{..............................(9)}$$

Differentiate with respect to $'X_7'$ we get,

$$1 = \frac{\mu_4 H_4 \left(-p_{47}\right) + \mu_5 H_5 \left(-p_{57}\right) + \mu_6 H_6 \left(-p_{67}\right) + \mu_7 H_7 \left(p_{75} + p_{76} + p_{77}\right)}{\mu_4 \left(-p_{47}\right) + \mu_5 \left(-p_{57}\right) + \mu_6 \left(-p_{67}\right) + \mu_7 \left(p_{75} + p_{76} + p_{77}\right)}$$

$$-\mu_4 H_4\, p_{47} - \mu_5 H_5 p_{57} + \mu_7 H_7 - \mu_6 H_6 p_{67} = -\mu_4 P_{47} - \mu_5 p_{57} - \mu_6 p_{67} + \mu_7 \qquad \text{..........................(10)}$$

Solving the equations (4), (5), (6), (7), (8), (9) & (10) we get

$$H_1 = 1 - \left\{ \frac{\lambda_1 B_a \left(1 - p_{32}.p_{23}\right) + \lambda_2 B_b \left\{ p_{21}\left(1 - p_{32}.p_{23}\right) + p_{23}\left(p_{31} + p_{32}.p_{21}\right) \right\} + \lambda_3 B_c \left(p_{31} + p_{32}.p_{21}\right)}{\mu_1 \left\{ \left(1 - p_{12}.p_{21}\right)\left(1 - p_{23}.p_{32}\right) - \left(p_{13} + p_{12} p_{23}\right)\left(p_{31} + p_{32}.p_{21}\right) \right\}} \right\}$$

$$\rho_1 = 1 - H_1$$

$$H_2 = 1 - \left\{ \frac{\lambda_1 B_a \left(p_{12} + p_{13}.p_{32}\right) + \lambda_2 B_b \left(1 - p_{13}.p_{31}\right) + \lambda_3 B_c \left\{ p_{31}\left(p_{12} + p_{13}.p_{32}\right) + p_{32}\left(1 - p_{13}.p_{31}\right) \right\} +}{\mu_2 \left\{ \left(1 - p_{13}.p_{31}\right)\left(1 - p_{32}.p_{23}\right) - \left(p_{12} + p_{13} p_{32}\right)\left(p_{21} + p_{31}.p_{23}\right) \right\}} \right\}$$

$$\rho_2 = 1 - H_2$$

$$H_3 = 1 - \left\{ \frac{\lambda_1 B_a \left\{ p_{12}\left(p_{23} + p_{13}.p_{21}\right) + p_{13}\left(1 - p_{12}.p_{21}\right) \right\} + \lambda_2 B_b \left(p_{23} + p_{13}.p_{21}\right) + + \lambda_3 B_c \left(1 - p_{21}.p_{12}\right)}{\mu_3 \left\{ \left(1 - p_{13}.p_{31}\right)\left(1 - p_{12}.p_{21}\right) - \left(p_{32} + p_{12} p_{31}\right)\left(p_{23} + p_{13}.p_{21}\right) \right\}} \right\}$$

$$\rho_3 = 1 - H_3$$

$$H_4 = 1 - \left\{ \frac{\mu_1 p_{14} \rho_1 + \mu_2 p_{24} \rho_2 + \mu_3 p_{34} \rho_3}{\mu_4} \right\}$$

$$\rho_4 = 1 - H_4$$

$$H_5 = 1 - \left\{ \frac{\left(1 - p_{76}.p_{67}\right)\left(p_{45} + p_{46}.p_{65}\right) + \left(p_{47} + p_{46}.p_{67}\right)\left(p_{75} + p_{76}.p_{65}\right)}{\mu_5 \left\{ \left(1 - p_{56}.p_{65}\right)\left(1 - p_{76}.p_{67}\right) - \left(p_{57} + p_{56}.p_{67}\right)\left(p_{75} + p_{76}.p_{65}\right) \right\}} \right\} \left(\mu_1 p_{14} \rho_1 + \mu_2 p_{24} \rho_2 + \mu_3 p_{34} \rho_3\right)$$

$$\rho_5 = 1 - H_5$$

$$H_5 = 1 - \left\{ \frac{\left(1 - p_{76}.p_{67}\right)\left(p_{45} + p_{46}.p_{65}\right) + \left(p_{47} + p_{46}.p_{67}\right)\left(p_{75} + p_{76}.p_{65}\right)}{\mu_5 \left\{ \left(1 - p_{56}.p_{65}\right)\left(1 - p_{76}.p_{67}\right) - \left(p_{57} + p_{56}.p_{67}\right)\left(p_{75} + p_{76}.p_{65}\right) \right\}} \right\} \left(\mu_1 p_{14} \rho_1 + \mu_2 p_{24} \rho_2 + \mu_3 p_{34} \rho_3\right)$$

$$\rho_5 = 1 - H_5$$

$$H_6 = 1 - \left\{ \frac{\left(1 - p_{75}.p_{57}\right)\left(p_{46} + p_{45}.p_{56}\right) + \left(p_{47} + p_{45}.p_{57}\right)\left(p_{76} + p_{75}.p_{56}\right)}{\mu_6 \left\{ \left(1 - p_{56}.p_{65}\right)\left(1 - p_{75}.p_{57}\right) - \left(p_{67} + p_{65}.p_{57}\right)\left(p_{76} + p_{75}.p_{56}\right) \right\}} \right\} \left(\mu_1 p_{14} \rho_1 + \mu_2 p_{24} \rho_2 + \mu_3 p_{34} \rho_3\right)$$

$$\rho_6 = 1 - H_6$$

$$H_7 = 1 - \left\{ \frac{\left(1 - p_{56}.p_{65}\right)\left(p_{47} + p_{46}.p_{67}\right) + \left(p_{45} + p_{65}.p_{46}\right)\left(p_{57} + p_{56}.p_{67}\right)}{\mu_7 \left\{ \left(1 - p_{67}.p_{76}\right)\left(1 - p_{56}.p_{65}\right) - \left(p_{75} + p_{76}.p_{65}\right)\left(p_{57} + p_{56}.p_{67}\right) \right\}} \right\} \left(\mu_1 p_{14} \rho_1 + \mu_2 p_{24} \rho_2 + \mu_3 p_{34} \rho_3\right)$$

$$\rho_7 = 1 - H_7$$

5. Fuzzification of the model

The arrival and service rates $\tilde{\lambda}_1, \tilde{\lambda}_2, \tilde{\lambda}_3$ & $\tilde{\mu}_1, \tilde{\mu}_2, \tilde{\mu}_3, \tilde{\mu}_4, \tilde{\mu}_5, \tilde{\mu}_6, \tilde{\mu}_7$ are pentagonal Intuitionistic fuzzy numbers (PIFN). The queue characteristics are indicated by the following formulas.

(i) Mean queue length

$$\tilde{L} = \tilde{L}_1 + \tilde{L}_2 + \tilde{L}_3 + \tilde{L}_4 + \tilde{L}_5 + \tilde{L}_6 + \tilde{L}_7$$

$$= \frac{\tilde{\rho}_1}{1-\tilde{\rho}_1} + \frac{\tilde{\rho}_2}{1-\tilde{\rho}_2} + \frac{\tilde{\rho}_3}{1-\tilde{\rho}_3} + \frac{\tilde{\rho}_4}{1-\tilde{\rho}_4} + \frac{\tilde{\rho}_5}{1-\tilde{\rho}_5} + \frac{\tilde{\rho}_6}{1-\tilde{\rho}_6} + \frac{\tilde{\rho}_7}{1-\tilde{\rho}_7}$$

(ii) Expected waiting time

$$\tilde{W} = \frac{\tilde{L}}{\tilde{\lambda}} \quad where\ \tilde{\lambda} = \tilde{\lambda}_1 + \tilde{\lambda}_2 + \tilde{\lambda}_3$$

(iii) Queue variance

$$\tilde{V} = \frac{\tilde{\rho}_1}{(1-\tilde{\rho}_1)^2} + \frac{\tilde{\rho}_2}{(1-\tilde{\rho}_2)^2} + \frac{\tilde{\rho}_3}{(1-\tilde{\rho}_3)^2} + \frac{\tilde{\rho}_4}{(1-\tilde{\rho}_4)^2} + \frac{\tilde{\rho}_5}{(1-\tilde{\rho}_5)^2} + \frac{\tilde{\rho}_6}{(1-\tilde{\rho}_6)^2} + \frac{\tilde{\rho}_7}{(1-\tilde{\rho}_7)^2}$$

6. Wingspans Fuzzy Ranking Method

The precise ranking of fuzzy numbers via the Wingspans method depends on the region between the slope of the membership function and the horizontal real axis. Assuming that $\tau(C)$ be a membership function and C_c be a core point of a fuzzy number $\tilde{C}$, respectively.

Wingspans centre of $\tilde{C}$ is $W_C = c_c - \frac{1}{2}\int_{-\infty}^{c_c} \tau_{\tilde{C}}(x)\,dx + \frac{1}{2}\int_{c_c}^{\infty} \tau_{\tilde{C}}(x)\,dx$

Let $\tilde{c} = [c_a, c_b, c_c, c_d, c_e]$ be a pentagonal fuzzy number, and our proposed new fuzzy ranking function for this number's Wingspans Centre is $W(\tilde{C}) = \frac{c_a + 2c_b + 2c_c + 2c_d + c_e}{8}$

Let $\tilde{C}^{PI} = \left(c_a, c_b, c_c, c_d, c_e; c_a^{'}, c_b^{'}, c_c^{'}, c_d^{'}, c_e^{'} \right)$ be a pentagonal intuitionistic fuzzy number. The wingspan centre of membership function of $\mu_{\tilde{C}^{PI}}(x)$ is defined by $W\left(\mu_{\tilde{C}^{PI}}(x)\right) = \frac{c_a + 2c_b + 2c_c + 2c_d + c_e}{8}$ & wingspan centre of non-membership function of $N_{\tilde{C}^{PI}}(x)$ is defined by $W\left(N_{\tilde{C}^{PI}}(x)\right) = \frac{c_a^{'} + 2c_b^{'} + 2c_c^{'} + 2c_d^{'} + c_e^{'}}{8}$ The w-center of $\tilde{C}^{PI}$ is $W(\tilde{C}^{PI}) = \frac{W\left(\mu_{\tilde{C}^{PI}}(x)\right) + W\left(N_{\tilde{C}^{PI}}(x)\right)}{2}$.

7. Numerical Analysis

Customers arrive in batches of fixed sizes $B_a = 2$, $B_b = 3$ and $B_c = 4$ with the steady-state probabilities $p_{12} = 0.3, p_{13} = 0.3, p_{14} = 0.4$; $p_{21} = 0.2, p_{23} = 0.3, p_{24} = 0.5$; $p_{31} = 0.2, p_{32} = 0.4, p_{34} = 0.4$; $p_{45} = 0.25, p_{46} = 0.35, p_{47} = 0.4$; $p_{55} = 0.3, p_{56} = 0.4, p_{57} = 0.3$; $p_{65} = 0.3, p_{66} = 0.3,$ $p_{67} = 0.4$; $p_{75} = 0.25, p_{76} = 0.35, p_{77} = 0.4$. The Pentagonal Intuitionistic fuzzy arrival and service rates and corresponding w-centers are tabulated as below:

Arrival rates	Service rates
$\tilde{\lambda}_1 = (0.95, 1, 2, 2.5, 3; 1, 1.5, 2, 3, 4) = 2.0594$	$\tilde{\mu}_1 = (33, 34, 35, 36, 37; 33.2, 34.2, 35, 36.4, 38) = 35.15$
$\tilde{\lambda}_2 = (1, 2, 3, 4, 5; 1.3, 2.5, 3, 4.1, 5.1) = 3.1$	$\tilde{\mu}_2 = (34, 35, 36, 37, 38; 34.2, 35.1, 36, 37.3, 39) = 36.125$
$\tilde{\lambda}_3 = (2, 3, 4, 5, 6; 2.1, 3.1, 4, 5.4, 6.5) = 4.1$	$\tilde{\mu}_3 = (35, 36, 37, 38, 39; 35.2, 36.2, 37, 38.5, 40) = 37.1625$
	$\tilde{\mu}_4 = (36, 37, 38, 39, 40; 36.5, 37.5, 38, 39.5, 41.5) = 38.25$
	$\tilde{\mu}_5 = (37.2, 38.1, 39, 40, 41; 37.4, 38.2, 39, 40.8, 42) = 39.2375$
	$\tilde{\mu}_6 = (38, 39, 40, 41, 42; 38.2, 39.2, 40, 41.4, 43) = 40.15$
	$\tilde{\mu}_7 = (39.2, 40.1, 41, 42, 43; 39.4, 40.2, 41, 42.8, 44) = 41.2375$

The calculated traffic intensities are $\tilde{\rho}_1 = 0.4214; \tilde{\rho}_2 = 0.6946; \tilde{\rho}_3 = 0.7634, \tilde{\rho}_4 = 0.7796,$ $\tilde{\rho}_5 = 0.6364, \tilde{\rho}_6 = 0.7885$ and $\tilde{\rho}_7 = 0.7779$, all of which are less than one.

The partial queue lengths are

$$\tilde{L}_1 = \frac{\tilde{\rho}_1}{1-\tilde{\rho}_1} = 0.7283$$

$$\tilde{L}_2 = \frac{\tilde{\rho}_2}{1-\tilde{\rho}_2} = 2.2744$$

$$\tilde{L}_3 = \frac{\tilde{\rho}_3}{1-\tilde{\rho}_3} = 3.2265$$

$$\tilde{L}_4 = \frac{\tilde{\rho}_4}{1-\tilde{\rho}_4} = 3.5372$$

$$\tilde{L}_5 = \frac{\tilde{\rho}_5}{1-\tilde{\rho}_5} = 1.7503$$

$$\tilde{L}_6 = \frac{\tilde{\rho}_6}{1-\tilde{\rho}_6} = 3.7281$$

$$\tilde{L}_7 = \frac{\tilde{\rho}_1}{1-\tilde{\rho}_1} = 3.5025$$

The mean queue length $\tilde{L} = \tilde{L}_1 + \tilde{L}_2 + \tilde{L}_3 + \tilde{L}_4 + \tilde{L}_5 + \tilde{L}_6 + \tilde{L}_7 = 18.7473$

Expected waiting time $\tilde{W}$ is 2.0247 and the Total queue variance $\tilde{V}$ is 76.6027.

Conclusion

In this work, we introduce an enlarged tri-cum biseries queue model. When compared to the prior model, it can shorten waiting times and queue lengths in an ambiguous environment. Our precious time and expense are reduced by the queuing technique offered in this research. The fuzzy queuing model designed here is employed in a various sectors to increase quality and productivity Also, we define a new formula for grading pentagonal Intuitionistic fuzzy numbers using wingspans ranking method. It more precisely and clearly rates the fuzzy numbers.

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