Original Article

Multi-Attribute Decision Making Problem Using OWA Operator

S. Vijaya Nirmala[1], K. Selvakumari[2*], S. Kavitha[3]

[1]Research Scholar, Vels Institute of Science, Technology and Advanced Studies (VISTAS), Chennai, Tamil Nadu, India. E-mail: nirmalashiva4@gmail.com

[2*]Professor, Vels Institute of Science, Technology and Advanced Studies (VISTAS), Chennai, Tamil Nadu, India. E-mail: selvafeb6@gmail.com

[3]Research Scholar, Vels Institute of Science, Technology and Advanced Studies (VISTAS), Chennai, Tamil Nadu, India. E-mail: kavithakavi.s1011@gmail.com

Abstract

The development of decision-making processes lies at core of the fuzzy set. Multi-attribute (MADM) is the most prevalent method of decision-making is of such situations. A common MADM problem involves analysing a comparison between the collection of alternatives and a selection of decision criteria. This paper provides a central government economic benefit to provinces and municipalities by using MADM situations.

Keywords: MADM, OWA Operator, Membership Function.

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Introduction

Multiple criteria decision making (MCDM) was developed by Benjamin Franklin (1772). MCDM has two various types that are Multi-attribute Decision Making (MADM) and Multi-Objective Decision Making (MODM) [6]. MODM model that investigates situations with a continuous choice space. Mathematical programming problems with many objective functions are a good illustration. Kuhn and Tucker are credited with the first mention of this problem, commonly known as the "vector-maximum" problem. [2]. MADM, on the other hand, concentrates on involving distinct decision regions. There is a fixed set of options for decision making in certain circumstances.

Even though MADM approaches vary greatly, many of them share key characteristics [1]. Numerous problems in the fields of information, finance, engineering, and other disciplines are resolved with OWA. The ordered weighted averaging (OWA) operator (OWAO) [9] is another fascinating aggregating operator that, despite being proposed in 1988, has not been frequently employed in the literature. The OWAO belongs to a group of parameterized aggregate operators that span the range of values high and low. Torra's [5] work includes the development of the weighted OWA (WOWA) operator, and Xu's [8] work includes the hybrid averaging (HA) operator. Both approaches arrived at unification between the OWA and the WA since both concepts were included in the formulation as exceptional circumstances. These models, however, appear to be a partial unification rather than a complete one, as shown in [3], despite being able to unify them, they are unable to analyse whether important these ideas are to the particular issue at hand. For instance, we might wish to give the OWA operator more weight in some situations because we think it's more important, and conversely. [4].

Preliminaries

Ordered Weighted Geometric (OWG) [7]

The $OWA: \mathbb{R}^q \to \mathbb{R}$, Then

$$OWA_\omega(\beta_1, \beta_2, \dots, \beta_q) = \sum_{l=1}^{n} \mu_l a_l$$

where a_l be a l^{th} highest set of arguments β_o $(o = 1,2,\dots,q)$ the arguments a_l $(l = 1,2,\dots,q)$ be ordered in descending order: $a_1 \geq a_2 \geq \cdots \geq a_l$, $\mu = (\mu_1, \mu_2, \dots, \mu_q)$ be a the weighting vector (WV) connected to the OWA function, $\mu_l \geq 0$, $l = 1,2,\dots,q$, $\sum_{l=1}^{p} \mu_l = 1$, and $\mathbb{R}$ will be collection of real numbers.

For example,

For $\mu = (0.1, 0.5, 0.4, 0.2)$ be a WV of OWA operator and (5, 14, 9, 20).

$$OWA_\mu(5, 14, 9, 20)$$
$$= 0.1 \times 20 + 0.5 \times 14 + 0.4 \times 9 + 0.2 \times 5$$
$$= 13.6$$

Theorem 2.1 [7]

Let $\mu_1 = \frac{1-\alpha}{p} + \alpha$, $\mu_i = \frac{1-\alpha}{p}$, $l \neq 1$ and $\alpha \in [0,1]$, then

$$\alpha OWA_{\mu^*}(\alpha_1, \alpha_2, \dots, \alpha_q) + (1-\alpha) OWA_{\mu_{Ave}}(\alpha_1, \alpha_2, \dots, \alpha_q) = OWA_{\mu}(\alpha_1, \alpha_2, \dots, \alpha_q)$$

If $\alpha = 0$, we get

$$OWA_{\mu_{Ave}}(\alpha_1, \alpha_2, \dots, \alpha_q) = OWA_{\mu}(\alpha_1, \alpha_2, \dots, \alpha_q)$$

If $\alpha = 1$, we get

$$OWA_{\mu^*}(\gamma_1, \gamma_2, \dots, \gamma_q) = OWA_{\mu}(\gamma_1, \gamma_2, \dots, \gamma_q)$$

Algorithm for MADM Problem Using OWA

Step 1: In MADM problem**,** $Z = \{z_1, z_2, z_3, \dots, z_p\}$ will be a finite collection of alternatives,

$O = \{o_1, o_2, o_3, \dots, o_q\}$ is a collection of attributes and its weight details is completely unspecified. A decision making calculates the alternatives z_i with respect to the attribute o_m and then we get the values b_{lm} $(l = 1,2,\dots,q; m = 1,2,\dots,p)$. $C = (c_{lm})_{p\times q}$.

Step 2: Generally, MADM problem have some ways of attributes namely

- Benefit
- Cost
- Fixed
- Deviation
- Interval
- Deviation interval

Each attribute values c_{lm} in the decision matrix $C = (c_{lm})_{q\times p}$ using the below formulas:

$$p_{lm} = \frac{c_{lm}}{\max_l \{c_{lm}\}}, \qquad l = 1,2,3,\dots,q, p \in L_1$$

$$p_{lm} = \frac{\max_l \{c_{lm}\}}{c_{lm}}, \qquad l = 1,2,3,\dots,q, p \in L_2$$

Hereafter we develop the decision matrix table of $P = (p_{lm})_{p\times q}$.

Step 3: The OWA aggregate every attribute value in p_{lm} $(m = 1,2,\dots,p)$ to the alternatives z_l. After get the attribute values by using definition 1.

$$z_l(\omega) = OWA_{\mu}(p_{l1}, p_{l2}, \dots, p_{lp}) = \sum_{l=1}^{q} \mu_m a_{lm}$$

Since a_{lm} are m^{th} highest value of p_{lm} $(m = 1,2,\dots,q)$, $\mu = (\mu_1, \mu_2, \dots, \mu_p)$ will be a weighting vector with OWA operator $\mu_p \geq 0, \sum_{m=1}^{p} \mu_m = 1$, by using theorem 1 or by the normal distribution.

$$\mu_m = \frac{e^{-\frac{(m-\sigma_p)^2}{2\omega_p^2}}}{\sum_{l=1}^{p} e^{-\frac{(m-\sigma_p)^2}{2\omega_p^2}}}, \qquad m = 1,2,\dots,p$$

Since $\sigma_p = \frac{1}{2}(1+p)$, $\omega_p = \sqrt{\frac{1}{p}\sum_{l=1}^{p}(l-\sigma_p)^2}$

Step 4: Every alternative rank z_l $(l = 1,2,3,\dots,q)$ corresponds to the values of $z_l(\mu)$ in descending order.

Numerical Example

Utilizing data from the China Industrial Economic Statistical Yearbook, evaluate the financial advantages of 16 cities and territories directly under the control of the federal government (2003). Given a collection of alternatives, choose one of the following: $Z = \{z_1, z_2, z_3, \dots, z_{16}\}$ = {Anhui, Beijing, Fujian, Guangdong, Hebei, Henan, Hubei, Hunan, Jiangsu, Jiangxi, Liaoning, Shandong, Shanghai, Shanxi, Tianjin, Zhejiang}. These indices used to evaluate the alternatives z_l $(l = 1,2,3,\dots,16)$ are given below: o_1: labour productivity of all staff (yuan per person); o_2: Profit each \$100 of sales revenue (yuan); o_3: capital interest tax rate (percent); o_4: profit rate of production (%), and o_5: circulating investment with a value of 100 yuan in factory output. o_5 is a price-type indicator among them, whereas others are profit-type indicator.

Alternatives	o_1	o_2	o_3	o_4	o_5
z_1	26,446	2.38	10.16	9.85	26.80
z_2	47,177	8.89	16.61	15.77	31.05
z_3	38,381	4.79	11.97	10.64	26.45
z_4	57,808	4.54	10.29	9.23	23.00
z_5	28,267	3.17	8.13	9.17	29.25
z_6	26,925	3.06	9.34	10.84	30.11
z_7	30,721	4.15	10.87	11.44	30.36
z_8	24,848	2.42	10.77	11.37	30.71
z_9	46,821	3.51	10.59	7.41	22.46
z_{10}	23,269	2.58	8.25	8.62	32.57
z_{11}	28,869	2.12	7.68	9.05	31.08
z_{12}	38,812	3.38	8.92	8.73	25.68
z_{13}	59,023	6.06	13.84	12.87	26.55
z_{14}	21,583	4.66	7.41	11.27	35.35
z_{15}	43,323	3.65	9.08	8.44	29.80
z_{16}	28,267	3.17	8.13	9.17	29.25

Alternatives	o_1	o_2	o_3	o_4	o_5
z_1	0.448	0.268	0.612	0.625	0.838
z_2	0.799	1.000	1.000	1.000	0.723
z_3	0.650	0.539	0.721	0.675	0.849
z_4	0.979	0.511	0.620	0.585	0.977
z_5	0.479	0.357	0.489	0.581	0.768
z_6	0.456	0.344	0.562	0.687	0.746
z_7	0.520	0.467	0.654	0.725	0.740
z_8	0.421	0.272	0.648	0.721	0.731
z_9	0.793	0.395	0.638	0.470	1.000
z_{10}	0.394	0.290	0.497	0.547	0.690
z_{11}	0.489	0.238	0.462	0.574	0.723
z_{12}	0.658	0.380	0.537	0.554	0.875
z_{13}	1.000	0.682	0.833	0.816	0.846
z_{14}	0.366	0.524	0.430	0.715	0.635
z_{15}	0.734	0.411	0.547	0.535	0.754
z_{16}	0.479	0.357	0.489	0.581	0.768

$\mu = (0.36, 0.16, 0.16, 0.16, 0.16)$ and $\alpha = 0.2$

$$z_1(\mu) = OWA_\mu(s_{11}, s_{12}, s_{13}, s_{14}, s_{15})$$
$$= 0.36 \times 0.838 + 0.16 \times 0.625 + 0.16 \times 0.612 + 0.16 \times 0.448 + 0.16 \times 0.268 = 0.6142$$

Similarly,

$z_2(\mu) = 0.9235$, $z_3(\mu) = 0.7192$, $z_4(\mu) = 0.7833$,
$z_5(\mu) = 0.5814$, $z_6(\mu) = 0.5964$, $z_7(\mu) = 0.6450$,
$z_8(\mu) = 0.5931$, $z_9(\mu) = 0.7274$, $z_{10}(\mu) = 0.5249$,
$z_{11}(\mu) = 0.5424$, $z_{12}(\mu) = 0.6556$, $z_{13}(\mu) = 0.8683$,
$z_{14}(\mu) = 0.6278$, $z_{15}(\mu) = 0.5814$, $z_{16}(\mu) = 0.7043$

From the before calculation of $z_l(\mu)$ for all $(l = 1,2,\ldots,16)$. We arrange the values in descending order

$$z_2 > z_{13} > z_4 > z_9 > z_3 > z_{16} > z_{12} > z_7 > z_{14} > z_1 > z_6 > z_8 > z_5 > z_{15} > z_{11} > z_{10}$$

RESULT

Based on above calculation results will be obtained that z_2 has the highest value and good alternative decision making.

CONCLUSION

The process of economic benefits from provinces and municipalities in the central government test involves more parameters. In this paper we use OWA operator to solve MADM problem to determining its benefit. In this way we can analysis more decision-making issues like company, electrician, product, stock market, etc.

REFERENCES

Chen S.J. and Hwang C.L. (1992), "Fuzzy Multiple Attribute Decision Making: Methods and Applications", Lecture Notes in Economics and Mathematical Systems, Springer-Verlag, Berlin, Germany.

Kuhn H.W. and Tucker A.W. (2014), "Nonlinear Programming", In Traces and emergence of nonlinear programming, pp. 247-258.

Merigó J.M. (2008), "New extensions to the OWA operators and its application in decision making", Ph.D. Thesis, Department of Business Administration, University of Barcelona.

Merigȯ J.M. (2009), "On the Use of the OWA Operator in the Weighted Average and Its Application in Decision Making", Proceedings of the World Congress on Engineering, 1, pp. 1-6.

Torra V. (1997), "The weighted OWA operator". International Journal of Intelligent Systems, 12, 153-166.

Triantaphyllou E., Shu B., Nieto Sanchez S. and Ray T. (1998), "Multi-Criteria Decision Making: An Operations Research Approach", Encyclopaedia of Electrical and Electronics engineering, 15, pp. 175-186.

Xu Z. (2015), "Uncertain Multi-Attribute Decision Making: Methods and Applications", Springer.

Xu Z.S. and Da Q.L. (2003), "An overview of operators for aggregating information", International Journal of Intelligent Systems, 18:953-968.

Yager R.R. (1988), "On ordered weighted averaging aggregation operators in multi-criteria decision making", IEEE Transactions on Systems, Man and Cybernetics B, 18, 183-190.

Ibrahim, S., & Koksal, M. E. (2021). Commutativity of sixth-order time-varying linear systems. Circuits, Systems, and Signal Processing, 40(10), 4799-4832.