Original Article

Numerical Investigation of Flow Over an Oscillating Circular Cylinder at Low Reynolds Number

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Abstract

A numerical investigation of flow over a transversely oscillating circular cylinder at a low Reynolds number ($75 \leq Re \leq 175$) is conferred in this paper. The open source CFD software, OpenFOAM is used to simulate the flow. The cylinder was made to oscillate transversely with frequency ratios ranging from 0.8 to 1.2 for different Reynolds numbers. Here, the frequency ratio (f_r) is the ratio of external excitation frequency (f_e) to that of Strouhal frequency (f_o). For all cases, the oscillation amplitude was constant and equal to 0.4. The Strouhal number and the aerodynamic coefficients were calculated for different frequency ratios and Reynolds numbers.

Keywords: Bluff Body Flows, Vortex Shedding, Strouhal Number, Coefficient of Drag, Coefficient of Lift.

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INTRODUCTION

When a bluff body with a forced oscillation is exposed to a flow stream, vortex shedding could be greatly altered. The topic has got an enormous number of applications in real life and has to be properly investigated. Numerous studies have already been made regarding the flow over a transversely oscillating cylinder. Experimental analysis by Bishop and Hassan [1], Koopmann [2] and Williamson and Roshko [3], numerical investigations by Blackburn and Henderson [4], Anagnostopoulos [5], [6], Lu and Dalton [7] are few to mention in which comprehensive studies have been conducted on this topic. Koopmann [2] has experimentally studied the consequences of forced oscillation of a circular cylinder on the wake characteristics at a low Reynolds number. The determination of the lock-in boundaries was also one of the main objectives. It was found that above a threshold amplitude, the lock-in phenomenon occurred. The lock-in phenomenon is the integration of the Strouhal frequency (shedding frequency) and the excitation frequency of the cylinder. When lock-in occurs, the Strouhal frequency (of a stationary cylinder) synchronizes and becomes equivalent to that of the external excitation oscillation frequency of the cylinder. The lock-in region's range increased with an increase in amplitude. Also, there were visible changes in the geometry of the wake due to the oscillation of the cylinder, which caused a reduction in the lateral spacing of wake vortices. Williamson and Roshko [3] have investigated experimentally, how large amplitude oscillations of circular cylinder impacts the wake synchronization. Various vortex synchronization regions were captured in a wavelength - amplitude plane. Also, it was found that below the critical trajectory wavelength, in each half-cycle, two like signed vortices merge and this formation creates a 2S mode of vortex. While, above the critical trajectory wavelength, a vortex pair is found to be convecting away from the centre line creating a 2P mode of vortices.

Recent numerical studies of Guilmineau and Queutey [8] captured the vortex patterns and vortex switching of a transversely oscillating circular cylinder and validated the experimental results carried out by Gu et al. [9] for a Reynolds number equal to 185. The oscillation amplitude was equal to 0.2 and the excitation frequency was varied between 0.8 and 1.2. It was found that as the frequency ratio increased, it produced a tighter vortex structure, and also, the vortex switching was observed to occur earlier than in the experimental study. Lu and Dalton [7] has also numerically investigated and compared their findings with Gu et al. [9] regarding the vortex switching phenomenon and also focusing on the aerodynamic coefficients. It was found that as the oscillation amplitude increases, the frequency ratio at which vortex switching occurs reduces. Furthermore, when Reynolds number was increased, the frequency ratio was found to decrease. Ajith Kumar et al. [10], [11] has discussed the effect of heating the circular cylinder has on the Strouhal number, the aerodynamic coefficients, and vortex shedding. It was found that the Strouhal number increases with heating. It was explained that, as flow over heated cylinder occurs, the vortices in the upper half of the cylinder are destabilized faster than that of the vortices in the lower half, which increases the shedding frequency. A correlation for Strouhal number in terms of Richardson number and Reynolds number was also proposed. It was also found that the coefficients of drag and lift decreased as the circular cylinder was heated.

Anagnostopoulos [5] has done a numerical examination of transversely oscillating circular cylinder focusing on the wake behavior, vortex timing and the hydrodynamic forces, at a fixed Reynolds number of 106. It was found from the study that, as the oscillation frequency is lesser or equal to the Strouhal frequency, a periodic wake pattern is observed for all amplitudes. But, when oscillation frequency is greater than the Strouhal frequency, the wake pattern is non-periodic. It was also found that for the frequency ratios less than 1, the lock-in region boundaries were on par with the experimental results of Koopmann [2], but for frequency ratios greater than 1, the lock-in occurs at amplitudes of oscillation greater than the amplitudes found in the experiment. This is due to the differences in the method of obtaining the lock-in boundary. In experimental studies, the lock-in region was established from the fluid velocity traces; but in numerical investigations, it was obtained from the hydrodynamic forces.

PROBLEM DEFINITION

The problem involves numerical investigation of flow ($75 \leq Re \leq 175$) over a circular cylinder. The cylinder is made to oscillate in a cross-flow direction with frequency ratio (f_r) ranging from 0.8 to 1.2 and oscillation amplitude (A_e/d) equal to 0.4 for all cases. The oscillation amplitude and other lengths corresponding to the domain are non-dimensionalized with diameter of the cylinder (d). The oscillation (f_e) is non-dimensionalized with the Strouhal frequency (f_o) of the stationary cylinder.

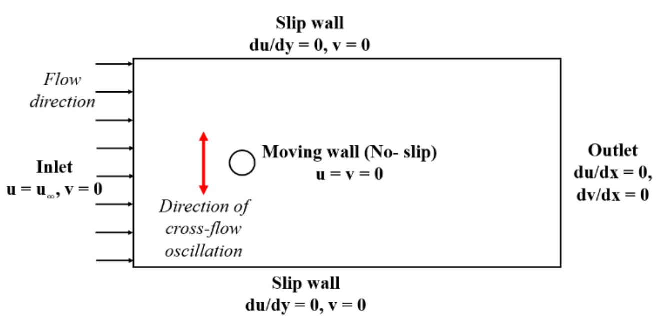

Fig. 1. Representational diagram of the domain with boundary conditions

The boundary conditions considered for the present simulation are illustrated in detail in Fig. 1. Here, u and v are components of fluid velocities in the x and y directions respectively. The top and bottom walls are considered to be slip walls while the circular cylinder is of no-slip condition. The cross-flow oscillation of the circular cylinder is represented using a double headed arrow and the flow direction is also represented using arrows while u_∞ is the inlet flow velocity.

NUMERICAL METHODOLOGY

The computational method requires the solving of the two–dimensional incompressible Navier-Stokes equation which is written in the non-dimensional form as below:

$$\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u} - \boldsymbol{w}) \cdot \nabla\boldsymbol{u} = -\nabla p + \frac{1}{Re}\nabla^2\boldsymbol{u}$$
$$\nabla . \boldsymbol{u} = 0$$

Where $\boldsymbol{u}$ is the fluid velocity, $\boldsymbol{w}$ is the mesh velocity, t is time, p is the fluid pressure, Re is the Reynolds number and ∇^2 is the Laplacian operator. The simulations were carried out in the open-source CFD software package, OpenFOAM. The pimpleFoam which is a solver for incompressible, turbulent flow of Newtonian fluids on a dynamic mesh is used for the numerical solution. The pimpleFoam algorithm is a consolidation of pisoFoam and simpleFoam algorithms. The simulations are run until the required convergence is attained.

The coefficients of drag and lift are given by,

$$C_d = \frac{2F_d}{\rho u_\infty^2 A}$$

$$C_L = \frac{2F_L}{\rho u_\infty^2 A}$$

where F_d and F_L are the drag and lift forces respectively, u_∞ is the freestream velocity, ρ is the fluid density, and A is the cross-sectional area. The Strouhal number is a non-dimensional representation of the shedding frequency which is given below.

$$St = \frac{f_o L}{u_\infty}$$

where, f_o is the Strouhal frequency, u_∞ is the freestream velocity while L is the characteristic length.

RESULTS AND DISCUSSION

A. Domain independence test

To find the optimum domain size for the problem, various domain sizes were created and tested. In all the domain sizes considered, the mesh density was kept constant, the Reynolds number selected is 100 and the cylinder is kept stationary. The various domains are chosen and corresponding C_d and St are calculated and consolidated in Table I. The domain independence test confirms that the domain of size $36d \times 24d$ with an upstream distance $8d$ is optimum.

TABLE I: Domain independence test: The values of $\overline{C_d}$ and St for Re = 100 for various domains. The upstream distance (L_u) is given by L_x / *4.5*. Lx is the length and L_y is the width of the domain. The percentage difference of the C_d and St values are indicated in the brackets in the corresponding columns.

S. No	L_x	L_y	L_u	C_d	St
1	21	14	4.67	1.4935	0.1770
2	24	16	5.33	1.4620 (2.15%)	0.1754 (0.91%)
3	27	18	6.00	1.4401 (1.52%)	0.1717 (2.15%)
4	30	20	6.67	1.4314 (0.61%)	0.1724 (0.41%)
5	33	22	7.33	1.4150 (0.85%)	0.1718 (0.35%)
6	36	24	8.00	1.4031 (0.85%)	0.1695 (1.36%)
7	39	26	8.67	1.3943 (0.63%)	0.1681 0.83%)

B. Grid independence test

The grid independence test was carried out on the optimum domain to choose the optimum number of cells that should be used for the problem. In the domain of size *36d × 24d* with an upstream distance of *8d*, the number of cells was varied and the values of the mean drag coefficient and Strouhal number were compared as can be seen in Table II.

TABLE II: Grid independence test: The values of $\overline{C_d}$ and St for Re = 100 for various meshes. The domain size is *36d × 24d*. The percentage difference of the C_d and St values are indicated in the brackets in the corresponding columns.

S. No	Number of cells	C_d	St
1	15049	0.3938	0.1626
2	19990	1.3946 (0.06%)	0.1653 (1.63%)
3	25035	1.3987 (0.29%)	0.1667 (0.84%)
4	30004	1.4072 (0.60%)	0.1709 (2.46%)
5	45022	1.4022 (0.36%)	0.1695 (0.83%)
6	50262	1.4001 (0.15%)	0.1695 (0.00%)
7	60140	1.4030 (0.21%)	0.1695 (0.00%)

We have selected an adaptive meshing closer to the cylinder and courser away from it. The optimum number of cells was found to be approximately 50000 (exactly 50262 cells).

C. Validation

After the selection of the optimum domain and grid, the domain was validated based on two research papers in the literature. The objective of the research by Anh-Hung et al. [12] was to numerically investigate the laminar flow over a circular cylinder which oscillated transversely to the freestream in which the Reynolds number considered was 185 for frequency ratios f_r ranging from 0.8 to 1.2 and oscillation amplitude from 0.2 to 0.5 was used. For validation, frequency ratios of 0.8, 0.9, 1.0 and 1.1 and oscillation amplitude of 0.4 are considered at Reynolds number 185.

TABLE III: Validation of $\overline{C_d}$ of the present study with Anh-Hung et al. [12] and Placzek et al. [13].

Re	f_r	A_e/d	C_d Present study	C_d Anh-Hung et al.	% Difference of C_d
185	0.8	0.4	1.47049	1.400	4.73
	0.9		1.64101	1.599	2.55
	1.0		1.83661	1.756	4.40
	1.1		1.61709	1.551	4.08
				Placzek et al.	
100	0.8	0.25	1.41591	1.405	0.75
	0.9		1.52764	1.496	2.05
	1.0		1.66088	1.630	1.84
	1.1		1.76840	1.748	1.15

A numerical investigation of the flow over a circular cylinder that oscillates transversely to the flow stream is discussed in the research work by Placzek et al. [13]. The Reynolds number was equal to 100 for forced transverse oscillation case in their study. The frequency ratios f_r ranging from 0.50 to 1.50 while the oscillation amplitude equal to 0.25 were considered in the work. For validation, the frequency ratios of 0.8, 0.9, 1.0 and 1.1 are considered at amplitude ratio 0.25 and Reynolds number 100. The results are matching quite well with these papers with an error of less than 5% always. The validation results are tabulated in Table III.

D. Aerodynamic coefficients

The aerodynamic coefficients are calculated for the various cases considered in the study and are discussed in the subsections below. The parameters considered are, Reynolds number, $75 \leq Re \leq 175$; frequency ratio, $0.8 \leq f_r \leq 1.2$ and oscillation amplitude $A_e/d = 0.4$.

Mean coefficient of drag ($\overline{Cd}$)

Fig. 2(a) shows the fluctuation of $\overline{C_d}$ with f_r (f_e/f_o) for various Re values. The trend of the plot between $\overline{C_d}$ and fr agrees well with those of Guilmineau and Quetey [8] and with those of Anh- Hung et al. [12].

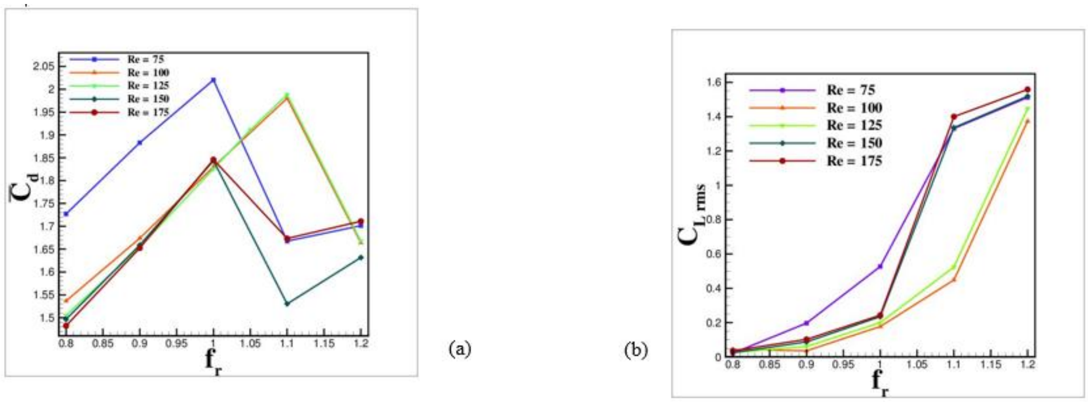

Fig. 2. Plots of (a) Mean coefficient of drag; (b) RMS values of coefficient of lift for various Re and frequency ratios chosen.

It can be observed from graph 2(a) that the maximum values of the mean coefficient of drag occurs at a frequency ratio of 1.0. It should be noted that, at f_r = 1.0, the excitation frequency becomes equivalent to the natural shedding frequency due to resonance. During resonance, flow separation takes place due to which the separation angle increases. When flow separation occurs, the pressure in the region behind the circular cylinder decreases due to which the pressure gradient increases. So, the drag forces also increase and attain the maximum value which can be seen in plot 2(a).

Coefficient of lift (C_{Lrms})

Fig. 2(b) shows the variation of C_{Lrms} with fr for various *Re* values. Unlike the mean coefficient of drag, the mean coefficient of lift is zero, despite the values of f_r and *Re*. The C_{Lrms} values are observed to be increasing with increase in frequency ratios as shown in Fig. 2(b) for all values of *Re*. The rms values of lift coefficient increased slowly and as the resonance phenomenon region is reached, a rapid jump is seen followed by a decrease in the rate of growth.

E. Strouhal number (St)

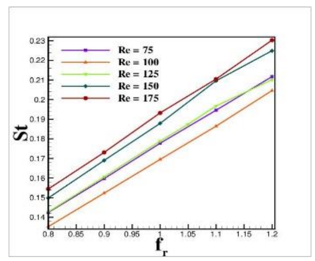

Fig. 3. The plot of St for various Re and frequency ratios considered for the study.

Fig. 3 shows the change of *St* with f_r for various *Re* values. It can be observed from Fig. 3 that *St* increases with f_r, almost in a linear manner. Fig. 4 represents the vorticity contours for various values of f_r at *Re* equal to 175. With an increase in frequency ratio, it can be noticed from Fig. 4 that the strength of vortices formed is decreasing. As the excitation frequency is increasing, an additional frequency due to the oscillation is also added to it which corresponds to an increase in Strouhal number as the frequency ratio is increasing.

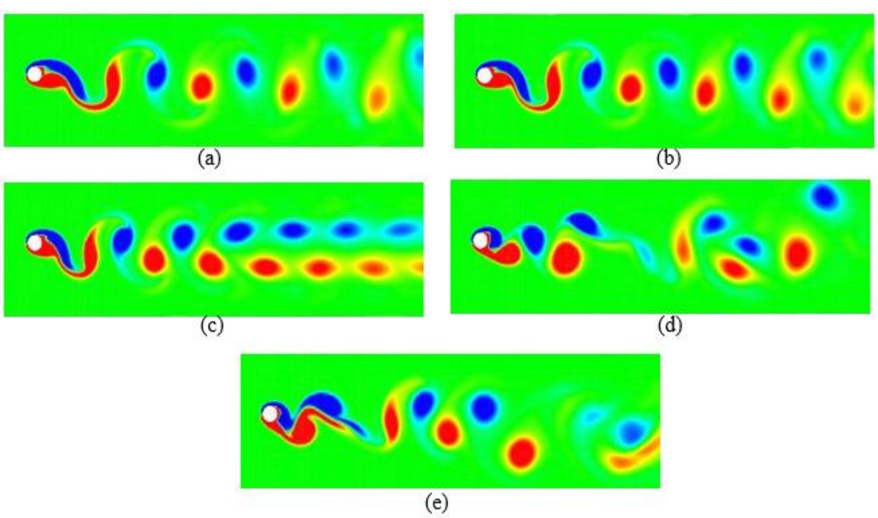

Fig. 4. Vorticity contours for frequency ratios (a) 0.8; (b) 0.9; (c) 1.0; (d) 1.1; (e) 1.2. Re = 175. The circular cylinder is at the upper most position in all cases.

A *St*-*Re* relationship for vortex shedding of a circular cylinder in laminar regime was given by Williamson [14] as below,

$$St = 0.00016(Re) + 0.1816 - \frac{3.3265}{Re} \quad (1)$$

Using the above *St* - *Re* relationship as a base, *St* is defined as a function of *Re* and f_r in the current case. A correlation of form (2) is made connecting *St*, *Re* and f_r.

$$St = 0.00016(Re) + 0.1816 - \frac{3.3265}{Re} + \boldsymbol{a(f_r)(Re^p) + b(f_r^q)} \quad \mathbf{(2)}$$

The values of the coefficients *a* and *b* and the powers *p* and *q* are also obtained from the curve fitting. The final equation is given by,

$$St = 0.00016(Re) + 0.1816 - \frac{3.3265}{Re} - 8.172(f_r)(Re^{0.003028}) + 8.298(f_r^{1.021}) \quad (3)$$

It can be observed from the above equation that the *St* - *Re* relation from [14] was used as such and the extra terms containing f_r (in bold letters) are added in (1) to get (2). The form (2) and hence (3) are formulated such that, at the limiting case of f_r tending to zero, (2) will become the same as (1), thus satisfying existing correlations of Williamson [14].

Conclusions

We have numerically investigated the two-dimensional laminar flow past a circular cylinder with cross-flow oscillation using an open-source CFD software package OpenFOAM. The Reynolds numbers (*Re*) considered in the study are 75, 100, 125, 150 and 175. The frequency ratios, $f_r = f_e/f_o$ considered in the current study are 0.8, 0.9, 1.0, 1.1 and 1.2 while the oscillation amplitude (A_e/d) was constant and equal to 0.4 for all the cases considered.

It was observed that for $f_r \leq 1$, the time plots of drag coefficient and lift coefficient were periodic while it wasn't periodic for $f_r > 1$. Next the behavior of aerodynamic coefficients such as $\overline{C_d}$ and C_{Lrms} are analyzed for various values of fr and are plotted. The general trend of the C_d vs f_r plot is such that, the value of C_d increases and a peak is obtained at $f_r = 1$ which corresponds to the resonance condition. The trend of the C_{Lrms} vs f_r plot implies that C_{Lrms} increases with f_r and there is a quick increase of C_{Lrms} starting at $f_r = 1$ and a non-linear trend is also observed. The *St* vs f_r plot shows a linear variation in general.

A correlation is proposed connecting *St*, *Re* and f_r as given below.

$$St = 0.00016(Re) + 0.1816 - \frac{3.3265}{Re} - 8.172(f_r)(Re^{0.003028}) + 8.298(f_r^{1.021})$$

The universal and continuous *St* - *Re* relationship given by Williamson [14] is modified by including *Re* and f_r and can be deduced to the original one for the non-oscillating case.

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