Original Article

On Some Homotopically Dense Subspaces of the Space $P(X)$ of Probability Measures Defined by an Infinite Metric Compact Set X

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Abstract

We study homotopically dense subspaces of the space $P(X)$ of probability measures defined by an infinite metric compact set that are finite-dimensional and infinite-dimensional topological manifolds. Considering various properties of subspaces of the space $P(X)$ of probability measures, the following are proved: 1) for any closed subset A of the compact set X other than X itself, there exists a strong deformation retraction $r: P(X) \to P(A)$; 2) for any infinite X and any of its closed subsets $A \subset X$ different from X itself, the subspace $P(A)$ is barycentrically open; 3) for any FAR compact set X, $P_{f,n}(X)$ is a FAR compact set; 4) for any infinite compact set X and any of its closed subsets $A \subset X$ different from X, the subspace $S_P(A) \setminus P(A)$ is homeomorphic to the Hilbert space ℓ_2; 5) for any infinite compact set X and any of its open subsets $A \subset X$ other than X, the subspace $S_P(X \setminus A) \setminus P(X \setminus A)$ is homeomorphic Hilbert space ℓ_2; 6) for any compact X the quotient space $P(X)|_{P_{f,n}(X)}$ is an ANR compact; 7) the projection $P: P(X) \to P(X)|_{P_{f,n}(X)}$ is a homotopy equivalence; 8) for any FAR compact set X the following conditions are equivalent:

a) the compact set $P_{f,n}(X)$ is shape dominated by some FAR compact set;

b) the compact set $P_{f,n}(X)$ has a point shape;

c) any mapping of $P_{f,n}(X)$ into an arbitrary ANR-compact space is homotopic to zero;

d) the compact set $P_{f,n}(X)$ is a fundamental retract of the Hilbert brick Q';

e) the compact set $P_{f,n}(X)$ is a movable compact set that is approximatively connected in all dimensions;

f) the compact set $P_{f,n}(X)$ is approximately connected in the class of all polyhedra;

i) the compact set $P_{f,n}(X)$ has a finite fundamental dimension and is approximately connected in all dimensions;

g) the compact set $P_{f,n}(X)$ is the inverse spectrum limit of FAR- compact sets;

j) the compact set $P_{f,n}(X)$ is cell-like, i.e. $P_{f,n}(X)$ can be cellularly embedded in Q;

k) There is a sequence of Hilbert bricks $Q'_n \subseteq Q$ such that $Q'_{n+1} \subseteq intQ_n$ and $\bigcap_{n=1}^{\infty} Q_n = P_{f,n}(X)$.

Keywords: Probability Measures, Manifolds, Homotopy Density, Fundamental Absolute Retract.

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INTRODUCTION

The space $P(X)$ of all probability measures of a compact set X is the set of all regular Borel probability measures on X equipped with the weakest of the topologies for which each $f_u : C(X) \to R$ functional taking the measure μ to $\mu(U)$ ($U-$ is an open set in X). It is known that the space $P(X)$ of probability measures of any infinite metric compact X is homeomorphic to the Hilbert cube $Q = I^{\aleph_0}$. It is also known that for any $\aleph_1 -$ degree of a non-one-point compact set K the space of probability measures $P(K^{\aleph_1})$ is homeomorphic to the Tikhonov cube $I^{\aleph_1}$ .i.e. $P(K^{\aleph_1}) \cong I^{\aleph_1}$, $I-$ segment [0,1]. Note, in particular, that all these spaces are topologically homogeneous. But for spaces $P(K^{\tau})$ for $\tau > \aleph_1$ the situation is different [1-3].

For an arbitrary compact set X and a measure $\mu \in P(X)$ its support $supp(\mu)$-is defined, which is the smallest of the closed sets $A \subset X$ for which $\mu(A) = \mu(X)$, i.e. $supp(\mu) = \bigcap\{A : A \subset X, A = \overline{A}, \mu \in P(A)\}$.

Let us single out a set for $n \in N$:

$$P_n(X) = \{\mu \in P(X) : \left|\sup p\mu\right| \le n\}$$

Note that $P_1(X)$; $\delta(X)$ - is the space of Dirac measures. Hence $\delta(X) \subset P_n(X)$.

$P_{\omega}(X) = \bigcup_{n=1}^{\infty} P_n(X)$ is the set of all finitely supported probability measures. Recall that the space $P_f(X) \subset P(X)$ consists of all probability measures [4-5].

$$\mu = m_1\delta(x_1) + m_2\delta(x_2) + ... + m_n\delta(x_n)$$

with finite supports, for each of which $m_i \ge \dfrac{n}{n+1}$ for some i.

For a natural number $n \in N$ we set:

$$P_{f,n}(X) = \{\mu \in P_f(X) : \left|\sup p\mu\right| \le n\}.$$

Obviously, for a metric compact set X and any $n \in N$ the sets $P_{f,n}(X)$ are closed in $P(X)$. It is obvious that $\delta(X) \subset P_{f,n}(X)$ and $P_f(X) = \bigcup_{n=1}^{\infty} P_{f,n}(X)$ i.e. $P_f(X) - \sigma -$ compact.

Therefore, the subspace $P_{\omega}(X) \subset P(X)$ and $P_{\omega}(X)$ is everywhere dense in $P(X)$ [6]. Therefore, the compact set $P_{f,n}(X)$ is the union of compact sets of the form $P_{f,i}(X)$ ($i < n$) i.e. $P_{f,n}(X) = \bigcup_{i=1}^{n} P_{f,i}(X)$. It is obvious that $P_{f,n}(X) \subseteq P_f(X)$ and $P_{f,n}(X) \subseteq P_{\omega}(X)$ [5-6].

Recall that a topological space Y is called an absolute /neighbourhood/ retract in the class K / is written $Y \in A(N)R(K)$ if $Y \in K$ and for every homeomorphism h, mapping Y onto a closed subset $h(Y)$ of the space X from the class K, the set $h(Y)$ is a retract /neighbourhood/ of the space X [7].

Definition [7]. A topological space X is called a manifold modeled on the space Y, or a $Y-$manifold, if every point in the space X has a neighborhood homeomorphic to an open subset of the space Y.

It follows from the Keller-Cayley results [2] that the space $P(X)$ of probability measures on an infinite compact set X is homeomorphic to the Hilbert cube Q, where $Q = \prod_{i=1}^{\infty}[-1,1]_i$ -hilbert cube. A Q -manifold is a separable metric space locally homeomorphic to the Hilbert cube Q, $W_i^{\pm} = \{(g_j) \in Q \mid g_i = \pm 1$ The $i-$th face of the Hilbert cube Q, $BdQ = \bigcup_{i=1}^{\infty} W_i^{\pm} -$ is called the pseudoboundary of Q, and $S = Q \setminus BdQ -$

pseudo-interior of Q cube [8].

The following objects play an important role in the theory of infinite-dimensional manifolds: a) the Hilbert cube Q; b) separable Hilbert space ℓ_2 ; c) $\sum$ is the line span of a standard brick Q' in the Hilbert space ℓ_2 where $Q' = \prod_{i=1}^{\infty}[-1, \frac{1}{2^i}]$. By the Anderson-Kadetz theorem, the Hilbert space ℓ_2 is homeomorphic to the pseudointerior S. It follows from Bessagui-Pelchinsky's results that $\sum$ is homeomorphic $\mathrm{r}intQ$[5]; 2) Here $\mathrm{r}intQ$ stands for the set $\{x=(x_n)\in Q \,|\, |x_n|<t<1$ for all $n\in N\}$. Further, $\mathrm{r}intQ \approx BdQ$ [5], hence $BdQ \approx \sum$; e) ℓ_2^f denotes a linear subspace of the Hilbert space ℓ_2, consisting of all points with only a finite number of coordinates different from zero; f) Q^f is a subspace Q of the Hilbert cube, consisting of all points, only a finite number of coordinates of which are different from zero. It is known that the spaces $Q, \sum$ and ℓ_2 are strongly infinite-dimensional, while the spaces ℓ_2^f and Q^f are weakly infinite-dimensional and these spaces are homogeneous [5].

The closed set *A* of the space *X* is called a Z-set in X [9] if the identity mapping id_X of the space X can be arbitrarily closely approximated by the mappings $f: X \to X \setminus A$.

A countable union of Z-sets in *X* is called a $\sigma - Z -$set in X [8].

Following [8], $\sigma - Z -$set B, of a Hilbert cube Q is called a boundary set in Q / denoted by $B(Q)$/ if $Q \setminus B \approx \ell_2$. More generally, a boundary set in a Q-manifold is a $\sigma - Z -$set whose complement is a $\ell_2 -$manifold.

It follows from the above that the pseudoboundary BdQ of the Hilbert cube Q is the boundary set for the Hilbert cube Q.

Let X be a topological space. A set $A \subset X$ is called homotopy dense in X [8] if there exists a homotopy $h(x,t): X \times [0,1] \to X$ such that $h(x,0) = id_X$ and $h(X \times (0,1]) \subset A$.

A the set $A \subset X$ is homotopy negligible in X [8] if $X \setminus A$ is homotopy dense in X.

An embedding $e: Y \to X$ is homotopy dense (respectively, homotopy negligible) if $e(Y)$ is a homotopy dense set (respectively, homotopy negligible) in X.

For each infinite compact set $X \in Comp$ and normal (or seminormal) functor $F: Comp \to Comp$ of infinite degree $\deg F = \infty[1]$, following M.M. [13], we accept the following notation:

a) $F_{\nabla}(X) = F(X) \setminus \eta_F(X),$ where $\eta_F : X \to F(X)$ is an embedding (identical) [3];

b) $F_{\nabla n}(X) = F(X) \setminus F_n(X),$ where $F_n(X) = \{a \in F(X) : |supp(a)| \le n\}$ and $\sup p(a)$ the support of the point $a \in F(X)$, for $n=1$ we identify $F_{\nabla_1}(X) \cong F_{\nabla}(X)$;

c) $F_{nk}(X) = F_n(X) \setminus F_k(X), n > k, n \ge 2$;

d) $F_{\omega}(X) = \bigcup_{n=1}^{\infty} F_n(X)$;

e) $F_{\nabla\omega}(X) = F(X) \setminus F_{\omega}(X)$;

f) $F_{\omega n}(X) = F_{\omega}(X) \setminus F_n(X)$.

Obviously, the subspace $F_{\nabla}(X)$ and $F_{\nabla n}(X)$ is open in $F(X)$, $F_{n,k}(X)$ is open in $F_n(X)$, $F_{\omega}(X)$ is a countable union of compact sets in $F(X)$ i.e. $F_{\omega}(X)$-- $\sigma-$ compact, on the other hand $F_{\omega}(X)$ is everywhere dense in $F(X)$; and $F_{\nabla\omega}(X)$ is $F_{\sigma}-$ a subspace of $F(X)$, the subspace of $F_{\omega n}(X)$ is an open set in $F_{\omega}(X)$.

The missing concepts and notation related to $F: Comp \to Comp$ functors can be found in [1-4,7-8].

MAIN PART

Let X be a zero-dimensional infinite compact set. Then $P(X);\ Q$.

Proposition 1. For any closed subset A of compact set X other than X itself, there exists a strong deformation retraction $r: P(X) \to P(A)$.

Proof. Let X be an infinite zero-dimensional compact $A \subset X$, $A \neq X$, $A = \overline{A}$. It is known that $P(A) \subset P(X)$ and $P(A)$ is a $Z-$set in $P(X)$ [7]. If A is a finite set, then it is known that $P(A);\ \sigma^{|A|-1}$

-simplex of dimension $|A|-1$. those. $P(A)$ is a $(|A|-1)$ -dimensional manifold. If the set A is infinite, then $P(A);\ Q$ and $P(A)\subset P(X)$. In any case, $P(A)$ is a compact AR contained in the Hilbert cube $P(X)$. It is known that a closed subset $A\subset X$ of a zero-dimensional compact set X distinct from X itself is a retract of X [3], i.e. there is a continuous retraction $r: X\to A$.

Consider a continuous mapping $P(r): P(X)\to P(A)$. It is known that the functor P preserves a retraction, so $P(r)$ will also be a retraction, i.e. the points of the set $P(A)$ remain fixed. Now we construct a deformation retraction $h(\mu,t): P(X)\times[0,1]\to P(A)$ by setting,

$$h(\mu,t)=(1-t)\mu+t\cdot r(\mu)$$

where $t\in[0,1],\ \mu\in P(X),\ r(\mu)=P(r)(\mu)$.

If $t=0$ then $h(\mu,0)=(1-0)\mu+0\cdot r(\mu)=\mu$. those. $h(\mu,0)=id_{P(X)}$.

If $t=1$, then $h(\mu,1)=(1-1)\mu+1\cdot r(\mu)=r(\mu)\in P(A)$. This means that $P(A)$ is a strong deformation retract. Proposition 1 is proved.

It follows from Proposition 1 and from the definition of barycentrically open sets [2].

Proposition 2. For any infinite X and any of its closed subsets $A\subset X$ other than X itself, the subspace $P(A)$ is barycentrically open.

A space X is called a fundamental absolute retract [7] (written: $X\in FAR$) if, for every space containing X as a closed subset, the set X is a fundamental retract of the space X.

A fundamental sequence $r=\{r_k: X, X\}_{MM}$ is called a fundamental retraction of the space X' on X (in M or in M) if $r_k(x)=x$ for each $x\in X$ and each $k=1,2,...$.

If there exists a fundamental retraction of the space X' to X in M, then X is called a fundamental retract of X' in M.

Let X and Y be two compact sets lying in the spaces M and N, respectively, where $M,N\in AR$ [7]. A sequence of mappings $f_k: M\to N$, where $k=1,2,...$ is called a fundamental sequence from X to Y if for each neighborhood V (under a neighborhood of the set Y is understood everywhere as a set whose open core contains Y, i.e. $A^0=X\setminus(\overline{X\setminus A})-$ inside-open core) of Y (in N) there exists a neighborhood U of a set in M such that the homotopy $\phi_k: U\times[0,1]\to V$, $\phi_k(x,0)=f_k(x)$ and $\phi_k(x,1)=f_{k+1}(x)$ for all $x\in U$ and for almost all k.

Theorem 1. For any FAR compact set X, $P_{f,n}(X)$ is a FAR compact set.

Proof. Let X be an arbitrary compact set. If X consists of a finite number of points, then there is a number $n\in N$ such that $P_{f,n}(X)$ contains a subset homeomorphic to $\sigma^{|X|-1}$ -simplex of dimension $|X|-1$. It is known that simplices of finite dimension are FAR -compact and, moreover, are AR compact. Now let X be an infinite compact space. It is known that $P(X);\ Q$. It was shown in [5] for any $n\in N$ that $P_f(X)$ is a $Z-$set in $P(X)$. On the other hand, the compact set $P_{f,n}(X)$ consists of linear combinations $m_1\delta_{x_1}+m_2\delta_{x_2}+...+m_n\delta_{x_n}$ the Dirac measure δ_{x_1}, where δ_{x_i} the Dirac measure of X at a point x_i, $0\le m_i\le 1$ and $\sum_{i=1}^n m_i=1$. those. $P_f(X)=\{\sum_{i=1}^n m_i\delta_{x_i}: m_i\ge 0, m_i\ le1, \sum_{i=1}^n m_i=1$, $\delta_{x_i}\in\delta(X)-$space of Dirac measures$\}$.

In this case, Theorem 1 [7, § 8.p.217] implies that $P_{f,n}(X)$ is FAR compact. Theorem 1 is proved.

Theorem 1 and Theorem 1 [7, § 8.str.217] imply

Corollary 1. For any FAR compact set X the following conditions are equivalent

1) $P_{f,n}(X)$ is shape-dominated by some FAR compact;

2) the compact set $P_{f,n}(X)$ has a point shape ;

3) Any mapping of the compact set $P_{f,n}(X)$ into an arbitrary ANR -compact set is homotopic to zero;

4) the compact set $P_{f,n}(X)$ is a fundamental retract of the Hilbert brick Q;

5) the compact set $P_{f,n}(X)$ is a movable compact set that is approximatively connected in all dimensions;

6) the compact set $P_{f,n}(X)$ is approximately

connected in the class of all polyhedra;

7) the compact set $P_{f,n}(X)$ has a finite fundamental dimension and is approximately connected in all dimensions;

8) the compact set $P_{f,n}(X)$ is the inverse spectrum limit of FAR-compact sets;

9) the compact set $P_{f,n}(X)$ is cell-like, i.e. $P_{f,n}(X)$ can be cellularly embedded in Q;

10) There is a sequence of Hilbert bricks $Q'_n \subseteq Q$ such that $Q'_{n+1} \subseteq intQ'_n$ and $\bigcap_{n=1}^{\infty} Q'_n = P_{f,n}(X)$.

From Theorem 8.1. [7] and Corollary 1 implies

Corollary 2. For any closed subsets A and B of an infinite compact set X, the following holds:

a) $P_{\nabla f}(A)$ and $P_{\nabla f}(B)$ are homeomorphic.

b) $P(X)|_{P_{f,n}(A)}$; $P(X)|_{P_{f,n}(B)}$.

Here $P(X)|_{P_{f,n}(A)}$ -factor is the space of the space $P(X)$ with respect to the subspace $P_{f,n}(A)$.

Note. From the result of [9] any compact X lying in the Hilbert space ℓ_2 is negligible in ℓ_2, then this $\ell_2 \setminus Y;\ \ell_2$.

$S_P(A)$ denotes the subspace $\{\mu \in P(X) : supp\mu \bigcap A \neq 0\}$ of the space $P(X)$.

It was shown in [5] that for any infinite compact set X and any closed subset $A \subset X$ different from X, the subspace $S_P(A)$ of the space $P(X)$ is homeomorphic Hilbert space ℓ_2. Therefore, from this fact and remarks, we obtain

Theorem 2. For any infinite compact set X and any of its closed subsets $A \subset X$ other than X, the subspace $S_P(A) \setminus P(A)$ is homeomorphic to the Hilbert space ℓ_2.

Theorem 3. For any infinite compact X and any of its open subsets $A \subset X$ other than X, the subspace $S_P(X \setminus A) \setminus P(X \setminus A)$ is homeomorphic to the Hilbert space ℓ_2.

Note that in [5-6] it was shown that for any infinite compact set X and a functor $P : Comp \to Comp$ of probability measures, subspaces:

1) $P_{\nabla}(X)$ is a Q-manifold;

2) $P_{\nabla P_f}(X) = P(X) \setminus P_f(X)$ is also a Q -manifold;

3) $P_{f,k}(X) - Q$-varieties if $X - Q -$ varieties;

4) $P_{\omega}(X)$ is a boundary set in $P(X)$;

5) $P_{\omega}(X)$ is a ℓ_2^f -manifold if X is finite-dimensional;

6) $P_{\omega}(X)$ is a $\sum$ -manifold if X is a Q -manifold;

7) $P_{\nabla\omega}(X)$ is ℓ_2 varieties;

8) $P_{\omega f}(X) = P_{\omega}(X) \setminus P_f(X)$ is ℓ_2^f -varieties if X is finite-dimensional;

Theorem 4. For any zero-dimensional infinite compact space *X* the space $P_{n,n-1}(X)$ is an $(n-1)$ manifold.

Proof. Let X be an infinite zero-dimensional metric compact set, then $P(X);\ Q$ and $\dim P_n(X) \leq n \dim X + \dim P_n(\tilde{n}) = \dim P_n(\tilde{n})$, where $\tilde{n}$ is an $n-$ point subset. Note that $P_n(\tilde{n}) = \sigma^{n-1} - (n-1)$ is a dimensional simplex [2-3]. Therefore, $\dim P_n(X) = n-1$. On the other hand, $P_n(X)$ is a linear combination of measures of the form: $m_1\delta_{x_1} + m_2\delta_{x_2} + \ldots + m_n\delta_{x_n}$ i.e. $P_n(X) = \{m_1\delta_{x_1} + m_2\delta_{x_2} + \ldots + m_n\delta_{x_n} : \sum_{i=1}^{n} m_i = 1, m_i \geq 0$ $\delta_{x_i} -$ Dirac measure at a point $x_i \in X\}$. Hence, for a compact set X, the space $P_n(X)$ consists of all closed $(n-1)$ dimensional simplices whose vertices lie at the points $[\delta_{x_1} + \delta_{x_2} + \ldots + \delta_{x_n}] = \sigma^{n-1}$. If we consider the space $P_n(X) \setminus P_{n-1}(X)$, then $P_n(X) \setminus P_{n-1}(X);\ P_{n,n-1}(X)$ is open in $P_n(X)$ since $P_{n-1}(X)$ is closed (compact) in $P_n(X)$.

Consider an arbitrary point (measure) $\mu \in P_{n,n-1}(X)$. Then the support $\sup p(\mu)$ of the measure μ consists of exactly n -points $\{x_1, x_2, \ldots, x_n\}$ m .e. $\mu = m_1\delta_{x_1} + m_2\delta_{x_2} + \ldots + m_n\delta_{x_n}, \sum_{i=1}^{n} m_i = 1, m_i > 0, m_i \neq 0$. This means that μ lies in $[\delta_{x_1}, \delta_{x_2}, \ldots, \delta_{x_n}]$ and is inner point of this closed simplex. those.

$$\mu \in int[\delta_{x_1}, \delta_{x_2}, \ldots, \delta_{x_n}] = [\delta_{x_1}, \delta_{x_2}, \ldots, \delta_{x_n}] \setminus Fr[\delta_{x_1}, \delta_{x_2}, \ldots, \delta_{x_n}]$$

We know that the interior of the closed simplex

$[\delta_{x_1},\delta_{x_2},...,\delta_{x_n}]$ is homeomorphic to $(n-1)$ dimensional space R^{n-1}. As an open neighborhood $O(\mu)$ we take $int[\delta_{x_1},\delta_{x_2},...,\delta_{x_n}]$. those. $O(\mu)=int[\delta_{x_1},\delta_{x_2},...,\delta_{x_n}]$. Hence, the space $P_{n,n-1}(X)$ is an $(n-1)$-manifold. Theorem 4 is proved.

If $n>1$ and X is infinite, then the set

a) $P_{f,2}(\tilde{2})$ consists of two segments:

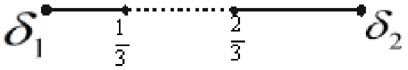

Figure 1.

b) $P_{f,3}(\tilde{3})$ consists of the following three pieces of an equilateral triangle:

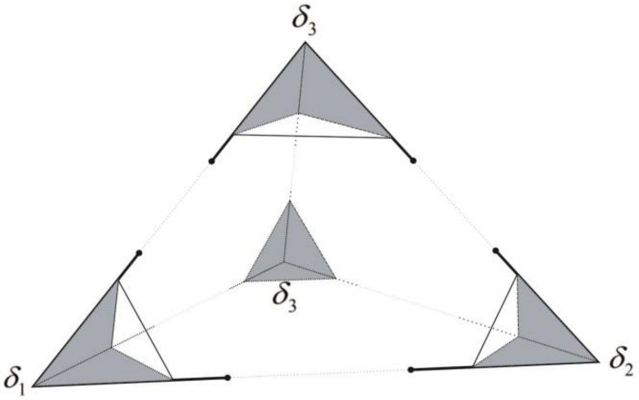

figure 2.

c) $P_{f,4}(\tilde{4})$ consists of the following pieces of a tetrahedron and then,

Note that the space $P_{(f,n)(f,n-1)}(X)=P_{f,n}(X)\setminus P_{f,n-1}(X)$ is an open subspace spaces $P_{n,n-1}(X)$. Therefore, $P_{(f,n)(f,n-1)}(X)$ is also an (n-1)-manifold. those. Occurs

Corollary 3. For any zero-dimensional infinite compact set X and $n>1$ the space $P_{f,n,n-1}(X)$ is an $(n-1)$-manifold.

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