Original Article

An Ill-Posed Problem For An Abstract Polycaloric Equation

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Abstract

Annotation: In article the incorrect task for abstract polycaloric the equation is studied and a stability assessment according to Tikhonov is given.

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Introduction

When solving problems of mathematical physics, an important role is played by the question of the correctness of the statement of the problem under study.

The concept of the correctness of the formulation of problems of mathematical physics was formulated at the beginning of our century by the famous French mathematician Hadamard.

A correct statement is usually understood as a statement of a problem that satisfies the following requirements:

I) The solution of the problem exists for all data belonging to some closed subspace in a normed linear space $C^{(K)}$, H, L_p, или $W_p^{(l)}$.

Most often this is the entire space•

2) The solution of the problem is unique in the same data class and in any similar class of solutions:

3) Infinitely small variations of these problems in $C^{(K)}$, H, L_p, or $W_p^{(l)}$ correspond to infinitesimals in some space $C^{(K)}$ or $W_p^{(l)}$ variations of the solution, that is, the solution, in a sense, continuously depends on the given problems.

Tasks that do not satisfy the listed requirements are called ill-posed.

Ill-posed, in the classical sense, problems have been encountered in the mathematical description of physical phenomena for a long time, but until relatively recently, these problems did not attract the serious attention of mathematicians.

Among mathematicians, it was believed that problems of this type are not associated with any physical phenomena, and therefore their study is of no interest.

The need to consider problems of mathematical physics that are incorrect in the classical sense in connection with the problems of interpreting geophysical observational data was first pointed out in the work of A.N. Tikhonov.

In the same work, new requirements were proposed for the formulation of problems of this type, which turned out to be completely natural from the point of view of the corresponding physical problems.

Later it turned out that classically ill-posed problems are encountered in the mathematical description of a wide variety of natural phenomena.

In the late fifties and especially in the early sixties, a number of new approaches appeared that became fundamental to the theory of ill-posed problems and attracted the attention of many mathematicians to it.

At present, there are a significant number of works devoted to the study of various problems of mathematical physics, formulated correctly according to Tikhonov.

Назовем задачу математической физики поставленной условно корректно или корректно по Тихонову, если выполняются следующие условия:

1) It is known a priori that the solution of the problem exists and belongs to some given set M of the functional space.

2) The solution of the problem on the set M is unique.

3) The solution of the problem on the set M continuously depends on the data, that is, infinitesimal variations of these problems that do not take the solution beyond the set M correspond to infinitesimal variations of the solution. The set M is called the correctness set, most often this set is compact.

In conditionally correct problems, existence theorems are not proved; existence is assumed to be known in advance, that is, it follows from experience in practice.

The uniqueness of the solution in the notion of conditional correctness does not differ from the classical notions of correctness.

One of the central points in the theoretical study of the conditionally well-posed problem is the proof of the uniqueness theorem.

The continuous dependence of the solution on the data on the correctness set also does not differ from the classical one, if we restrict ourselves to considering only the correctness set.

When we consider a compact set as M, then in this

case, it turns out that the continuous dependence of the solution on the data on the well-posedness set follows from the uniqueness of the solution.

It is possible to establish for some problems not only the correctness according to Tikhonov, but also a specific stability estimate. However, the proof of Tikhonov's theorem does not contain indications of how to obtain specific functions for specific problems.

In the works of M.M. Lavrent'ev, we obtain stability estimates for the Cauchy problem for the heat equation with inverse time.

Consideration of a problem that is ill-posed in the classical sense as a conditionally correct one makes it possible to construct an approximate solution with guaranteed accuracy based on approximate data using the enumeration method.

Some ill-posed problems, when the number M is known, were considered in the works of M.M. Lavrentiev. However, for a number of applied problems, a situation is typical when M is unknown. Let there be a problem of mathematical physics that is ill-posed in the classical sense.

In the present article, some new abstract ill-posed problems for the polycaloric equation are posed and studied. Ill-posed problems for second-order differential equations have been studied in many works by both Soviet and foreign authors.

Recently, ill-posed problems for differential equations of higher order have been intensively studied.

Considered in the classical sense, that is, for all the problems under consideration, there is no continuous dependence of the solution on the data. Let us investigate the conditionally correctness and efficiency of various methods of regularization of some new abstract ill-posed problems for a polycaloric equation.

The following problem is considered. It is required to find a solution to an abstract polycaloric equation,

$$K_{+}^{n}u(t) \equiv \left(\frac{d}{dt}+A\right)^{n} u(t)=0, \quad 0<t<T, \tag{1}$$

satisfying the following boundary conditions:

$$\begin{cases} u\big|_{t=l} = u(l), \\ \dfrac{du}{dt}\Big|_{t=l} = u'(l), \\ \dots\dots\dots\dots\dots\dots \\ \dots\dots\dots\dots\dots\dots \\ \dfrac{d^{(n-1)}u_{(t)}}{dt}\Big|_{t=l} = u_{(l)}^{(n-1)} \end{cases} \tag{2}$$

where $u(t)$ - abstract function with values in Hilbert space - constant, positive-definite, self-adjoint, linear, unbounded with everywhere dense domain of definition. $\left(D\left(A^{n}\right), DCH\right)$ operator operating from H in H, and $u(l), u'(l), u''(l), \dots u^{(n-1)}(l) \in H.$

Theorem 1. If $u_1, u_2, ...u_n$ - solution of the caloric equation, then the function $u = u_1 + (t-l)u_2 + (t-l)^2 u_3 + ...(t-l)^{n-1} u_n$ there is a solution to (1) and vice versa, for each given abstract polycaloric function u there are such functions $u_1, u_2, ...u_n$, that there is an equality

$$u(t) = \sum_{k=1}^{n} (t-l)^{k-1} u_k(t) \qquad (3)$$

Using representation (3), the solution of problem (1) - (2) can be reduced to the solution of the following n tasks:

$$\begin{cases} \frac{du_1}{dt} + Au_1 = 0, \\ u_1\big|_{t=l} = u_1(l), \end{cases} \quad \begin{cases} \frac{du_2}{dt} + Au_2 = 0, \; .... \\ u_2\big|_{t=l} = u_2(l), \; .... \end{cases} \quad \begin{cases} \frac{du_n}{dt} + Au_n = 0, \\ u_n\big|_{t=l} = u_n(l), \end{cases} \qquad (4)$$

where

$$u_1(l) = u(l)$$

$$u_2(l) = u'(l) + Au(l)$$

$$u_3(l) = \frac{1}{2}\left(u''(l) - 2Au(l) + A^2 u(l)\right)$$

....

....

$$u_n(l) = \sum_{k=1}^{n} \sum_{i=0}^{n-1} C_{k-1}^{i} A^{k-i} u^{i}(l)$$

$$C_{k-1} = \frac{1}{(k-1)!}$$

These problems are ill-posed in the classical sense. We will study them for conditional correctness according to Tikhonov [1]. The Cauchy problem for equation (1) is considered and studied in [2]. The multipoint problem for equation (1) was studied in [3].

Theorem 2. For any solution of problem (1) - (2)

$$\|u(t)\|_H \le \sum_{k=1}^{n} \sum_{i=1}^{k} C_{k-1} \begin{cases} \left(\|u^{(i)}(l)\| \cdot \|A^{k-i}\|\right)^{\frac{t}{l}} \cdot \|K_+^{i-1} u(0)\|^{1-\frac{t}{l}}, & 0 \le t \le l, \\ \|u(l)\|, & l \le t \le T \end{cases} \qquad (5)$$

Note that inequality (5) implies the uniqueness of the solution to problem (1)–(2) and the conditional well-posedness of this problem in the class

$$\left\{u : \|K_+^{n-1} u(0)\| \le M\right\}.$$

This theorem is proved by the logarithmic convexity method [1].

The paper uses the methods of the theory of conditionally well-posed problems for differential-operator equations, developed in the works of A.N. Tikhonova, M.M. Lavrentiev, V.K. Ivanov, Lattes and J. Lions and others.

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