Original Article

Thermal radiation effect of mixed convection of couple stress fluid in a vertical channel in the presence of heat source or heat sink

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DOI: 10.47750/pnr.2022.13.S06.251

Abstract

The fully developed mixed convection of couple stress fluid in a vertical channel in the presence of heat source or sink and thermal radiation effect is analyzed. The two boundaries of the channel are considered as isothermal-isothermal, isoflux-isothermal and isothermal-isoflux for the left and right walls and kept either at equal or at different temperature. The governing momentum and energy equations are coupled and nonlinear due to the viscous effects. The results are represented graphically for different values of buoyancy parameter , couple stress parameter 'a' and radiation parameter F on velocity and temperature distributions. We observe that for purely viscous fluid with the flow reversal was at the hot wall whereas for couple stress fluid there is a flow reversal both at left and right walls. The effect of on the flow for couple stress fluid is dominating compared to viscous fluid both on velocity and temperature. The profiles of temperature are significant for couple stress fluid for different values of whereas the profiles were not sensible for different values of for viscous fluid.

INTRODUCTION

The study of non-Newtonian fluids has received much attention due to their many practical applications in medical sciences, engineering and technology, such as liquid crystals, fluid film lubrication etc. In the category of non-Newtonian fluids, couple stress fluid has distinct features such as polar effects in addition to possessing large viscosity. The consideration of couple stress in addition to Cauchy stress has led to the recent development of several theories of fluid micro-continua.

The couple stress fluid model has wide applications in bio-fluids, colloidal fluids and in engineering for pumping fluids such as synthetic lubricants. Studies on natural convection in a vertical channel with non-Newtonian fluid are relatively sparse compared to the problem with Newtonian fluid. Studies related to channel flow and heat transfer characteristics of couple-stress fluids not only present theoretical problem challenges, but also find several applications in many industrial processes such as the extrusion of polymer fluids, solidification of liquid crystals, cooling of metallic plates in a bath, colloidal solutions, etc. These fluids are capable of describing various types of lubricants, blood and suspension fluids. Stokes (1966) introduced the theory of couple stress fluids. The main feature ofcouple stresses is to introduce a size dependent effect. Classical continuum mechanics neglects the size effect of material particles within the continua. However, in some important cases such as fluid flow with suspended particles, this cannot be true and a size dependent couple-stress theory is needed. The spin field due to micro-rotation of freely suspended particles sets up an anti-symmetric stress, known as couple-stress, and thus forms a couple-stress fluid. A review of couple stress fluid dynamics was reported by Stokes (1984). Cheng, Kou (1990) reported flow reversal and heat transfer of fully developed mixed convection in vertical channels. Lavine (1988) studied the fully developed opposing mixed convection between inclined parallel plates. Umavathi and Malashetty (1999) analyzed the effects of couple stresses on Oberbeck convective flow in a vertical porous channel and it is noted that both the porous parameter and the couple stress parameter suppress the flow.The problem of steady laminar fully developed flow and heat transfer in a horizontal channel consisting of a couple-stress fluid sandwiched between two clear viscous fluids is analyzed analytically by J.C Umavathi, A J Chamkha (2005). The fully developed mixed convective flow of couple stress fluid in a vertical channel in the presence of heat generation or absorption is analyzed by Patil Mallikarjun (2015) in which it is proved that the perturbation parameter has dominating influence on the flow of couple stress fluid compared to viscous fluids. Mixed convection of couple stress permeable fluid in a vertical channel in the presence of heat generation or heat absorption is again analyzed by Patil Mallikarjun (2015) and has proved that the flow and the temperature are suppressed by the influence of porous parameter on the couple stress fluid. The electrically conducting flow of couple stress fluid in a vertical porous layer is investigated by Sreenadh, Nanda Kishore (2011).Heat and mass transfer effects on an unsteady MHD flow of a couple-stress fluid in a horizontal wavy porous space with travelling thermal waves in the presence of a heat source and viscous dissipation has been studied by Muthuraj, Srinivas (2013).

In spite of the various applications of non-Newtonian fluids much work has not been found in the literature on mixed convection flows. Hence it is the objective of this paper to study the problem of mixed convection couple stress fluid in a vertical channel in the presence of heat source or heat sink and thermal radiation effect.

Mathematical Formulation: The Oberbeck- Boussenisq approximation is

$$\rho = \rho_0\left(1-\beta\left(T-T_0\right)\right) \quad (1)$$

The momentum balance equation for couple stress fluid is,

$$g\beta\left(T-T_0\right)-\frac{1}{\rho_0}\frac{dP}{dX}+\frac{\mu}{\rho_0}\frac{d^2U}{dY^2}-\frac{\eta}{\rho_0}\frac{d^4U}{dY^4}=0 \quad (2)$$

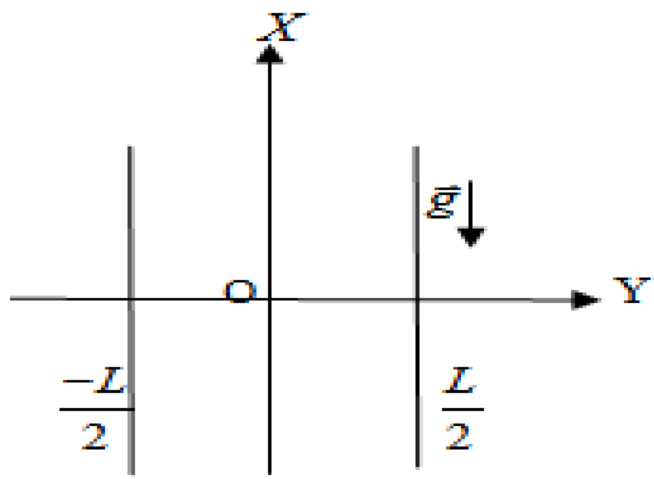


Figure 1- Drawing of the parallel plate and rectangular coordinate axes

The Y-momentum balance equation can be expressed as $\frac{dP}{dY}=0$ (3)

where $P = p + \rho_0 g X$ is the difference between the pressure and the hydrostatic pressure. The temperature is T_1, at the left wall $Y=-L/2$ and the temperature is T_2, at the right wall $Y=-L/2$, with $T_2 \ge T_1$. These conditions are compatible with equation (2) only when dP/dX is independent of X. Hence, there exists a constant A such that,

$$\frac{dP}{dX}=A \quad (4)$$

Solving the equations (2) and (3), we obtain

$$\frac{\partial T}{\partial X}=0 \quad (5)$$

which implies that the temperature also depends on Y .By considering the effects of viscous dissipation, along with heat source or sink, the energy balance equation reduces to

$$\alpha \frac{d^2T}{dY^2} + \frac{\mu}{\rho_0 C_p}\left(\frac{dU}{dY}\right)^2 \pm \frac{Q(T-T_0)}{\rho_0 C_p} - \frac{1}{\rho_0 C_P}\frac{dq_R}{dY} = 0 \tag{6}$$

Simplifying the equations (2) and (6) allows one to obtain a non-linear differential equation for U in the form,

$$\frac{d^6U}{dY^6} = \left(\frac{\mu}{\eta} \mp \frac{Q}{K}\right)\frac{d^4U}{dY^4} \pm \frac{\mu Q}{K\eta}\frac{d^2U}{dY^2} - \frac{\mu \rho_0 \beta g}{K\eta}\left(\frac{dU}{dY}\right)^2 \mp \frac{QA}{K\eta} + \frac{\rho_0 g \beta}{K\eta}\frac{dq_R}{dY} \tag{7}$$

The boundary conditions on U are both the no slip conditions

$$U = \frac{d^2U}{dY^2} = 0 \quad \text{at} \quad Y = \pm\frac{L}{2} \tag{8}$$

$$\frac{d^4U}{dY^4} = -\frac{A}{\mu} + \frac{\beta g \rho_0 (T_1 - T_0)}{\eta} \text{ at } \quad Y = -\frac{L}{2}$$

$$\frac{d^4U}{dY^4} = -\frac{A}{\mu} + \frac{\beta g \rho_0 (T_2 - T_0)}{\eta} \text{ at } \quad Y = \frac{L}{2} \tag{9}$$

The following quantities are employed for writing equations (6) to (9) in the dimensionless form

$$u = \frac{U}{U_0}; \quad \theta = \frac{T-T_0}{\Delta T}; \quad y = \frac{Y}{D}; \quad Gr = \frac{g\beta \Delta T D^3}{\nu^2}; \; k = \frac{\eta}{\mu D^2}; \; a^2 = \frac{1}{k}$$

$$\lambda = \frac{Gr}{\mathrm{Re}}; R_T = \frac{T_2 - T_1}{\Delta T}; \mathrm{Re} = \frac{U_0 D}{\gamma}; \mathrm{Pr} = \frac{\gamma}{\alpha}; Br = \frac{\mu U_0^2}{K\Delta T}; \phi = \frac{QD^2}{K}; F^2 = \frac{CD^2}{K}$$

$$A = \frac{-48\mu U_0}{D^2}; \frac{dq_R}{dY} = c(T - T_0); \sigma^2 = \frac{D^2}{K}; \alpha = \frac{K}{\rho_0 C_P}; \gamma = \frac{\mu}{\rho_0} \tag{10}$$

Thereference velocity U_0 and the reference temperature T_0 are, $U_0 = -\frac{AD^2}{48\mu}$; $\quad T_0 = \frac{T_1 + T_2}{2}$.

The temperature difference ΔT is given by

$$\Delta T = T_2 - T_1 \quad \text{if} \quad T_1 < T_2 \quad \text{or by} \tag{11}$$

$$\Delta \mathrm{T} = \frac{\gamma^2}{C_p D^2} \quad \text{if} \quad T_1 = T_2 \tag{12}$$

The dimensionless form of equations (6) to (9) are as follows,

$$\frac{d^2\theta}{dy^2} = -Br\left(\frac{du}{dy}\right)^2 \mp (\phi - F^2)\theta \tag{13}$$

$$\frac{d^6u}{dy^6} = \left(a^2 \mp \phi + F^2\right)\frac{d^4u}{dy^4} + a^2(\pm\phi - F^2)\frac{d^2u}{dy^2} - \lambda Bra^2\left(\frac{du}{dy}\right)^2 + 48(\pm\phi - F^2)a^2$$

$$u = \frac{d^2u}{dy^2} = 0 \quad \text{at} \quad \frac{1}{4} \tag{14}$$

$$\frac{d^4u}{dy^4} = 48a^2 - \frac{R_T \lambda a^2}{2} \quad \text{at} \quad y = -\frac{1}{4} \tag{15}$$

$$\frac{d^4u}{dy^4} = 48a^2 + \frac{R_T \lambda a^2}{2} \quad \text{at} \quad y = \pm\frac{1}{4} \tag{16}$$

Temperature field can also be obtained while substituting equations (10) and (11) in momentum equation (2) one obtains,

$$\theta = -\frac{1}{\lambda}\left(48 + \frac{d^2u}{dy^2} - \frac{1}{a^2}\frac{d^4u}{dy^4}\right) \tag{17}$$

Analytical solution by Perturbation method: Equation (14) along with boundary conditions given by (15) and (16) is nonlinear and hence it is difficult to find closed form solution. However, to obtain an analytical solution of equation (14), we employ the perturbation method using the dimensionless parameter $\varepsilon\,(<< 1)$, which is defined as

$$\varepsilon = Br\ \lambda = \mathrm{Re}\ \mathrm{Pr}\ \frac{\beta\, g\, D}{C_p} \tag{18}$$

Then the temperature field is obtained using equation (17). Thus, we assume that u can be expressed as

$$u(y) = u_0(y) + \varepsilon\ u_1(y) + \varepsilon^2\ u_2\ (y) + \cdots\cdots\cdots = \sum_{n=0}^{\infty} \varepsilon^n\ u_n(y) \tag{19}$$

The second and higher order terms of ε gives a correction to u_0 and θ_0 accounting for the viscous and Darcy dissipation effects. Substituting equation (19) in equation (14) and equating coefficients of like powers of ε to zero, one obtains the boundary value problem for $n = 0$ and $n = 1$ as,

Isothermal-Isothermal case (T_1-T_2):

$$\frac{d^6u_0}{dy^6} = \left(a^2 \mp \phi + F^2\right)\frac{d^4u_0}{dy^4} \pm a^2(\phi - F^2)\frac{d^2u_0}{dy^2} \pm 48a^2(\phi - F^2) \tag{20}$$

$$\frac{d^6u_1}{dy^6} = \left(a^2 \mp \phi + F^2\right)\frac{d^4u_1}{dy^4} \pm a^2(\phi - F^2)\frac{d^2u_1}{dy^2} - a^2\left(\frac{du_0}{dy}\right)^2 \tag{21}$$

for the case of heat generation or absorption respectively. The boundary conditions of these equations are

$$u_0 = 0 \quad \text{at} \quad y = \pm\frac{1}{4} \tag{22}$$

$$\frac{d^2u_0}{dy^2} = 0 \quad \text{at} \quad y = \pm\frac{1}{4} \quad \text{and} \quad \frac{d^4u_0}{dy^4} = 48a^2 \pm \frac{R_T\lambda a^2}{2} \quad \text{at} \quad y = \pm\frac{1}{4} \tag{23}$$

$$u_1 = 0 \ \text{at} \quad y = \pm\frac{1}{4} \tag{24}$$

$$\frac{d^2u_1}{dy^2} = 0 \ \text{at} \quad y = \pm\frac{1}{4}$$

$$\frac{d^4u_1}{dy^4} = 0 \quad \text{at} \quad y = \pm\frac{1}{4} \tag{25}$$

Equations (20) and (21) are ordinary linear differential equations and their exact solutions can be easily found. Evaluation of exact solution for $n = 2$ becomes very tedious and hence neglecting the terms for $n = 2$ the solution is obtained up to terms of $0(\varepsilon^1)$ to get, $u_0 = C_1 + C_2 y + C_3 Cosha\, y + C_4 Sinha\, y + C_5 Cos(\sqrt{\phi - F^2})y + C_6 Sin(\sqrt{\phi - F^2})y - 24y^2$ (26)

for the case of heat generation and

$$u_0 = C_1 + C_2 y + C_3 Cosha\, y + C_4 Sinha\, y + C_5 Cosh(\sqrt{\phi + F^2})y + C_6 Sinh(\sqrt{\phi + F^2})y - 24y^2 \qquad (27)$$

for the case of heat absorption.

$$\begin{aligned}
u_1 = {} & C_7 + C_8 y + C_9 Coshay + C_{10} Sinha\, y + C_{11} Cos(\sqrt{\phi - F^2})y + \\
& C_{12} Sin(\sqrt{\phi - F^2})y + l_1 Cosh2ay + l_2 Cos(2\sqrt{\phi - F^2})y + l_3 Sinh2ay + \\
& l_4 Sin2(\sqrt{\phi - F^2})y + l_5 Coshay\ Cos(\sqrt{\phi - F^2})y + \\
& l_6 Sinhay\, Sin(\sqrt{\phi - F^2})y + l_7 Coshay\, Sin(\sqrt{\phi - F^2})y + \\
& l_8 SinhayCos(\sqrt{\phi - F^2})y + l_9 yCoshay + l_{10} ySinha\, y + \\
& l_{11} y^2 Coshay + l_{12} y^2 Sinha\, y + l_{13} y^2 Cos(\sqrt{\phi - F^2})y + l_{14} y^2 Sin(\sqrt{\phi - F^2})y + \\
& l_{15} yCos(\sqrt{\phi - F^2})y + l_{16} ySin(\sqrt{\phi - F^2})y + l_{17} y^4 + l_{18} y^3 + l_{19} y^2
\end{aligned} \qquad (28)$$

for the case of heat generation and

$$\begin{aligned}
u_1 = {} & C_7 + C_8 y + C_9 Coshay + C_{10} Sinha\, y + C_{11} Cosh(\sqrt{\phi + F^2})y + \\
& C_{12} Sinh(\sqrt{\phi + F^2})y + l_1 Cosh2ay + l_2 Cosh(2\sqrt{\phi + F^2})y + l_3 Sinh2ay + \\
& l_4 Sin2(\sqrt{\phi + F^2})y + l_5 Cosh(a + \sqrt{\phi + F^2})y + l_6 Cosh(a - \sqrt{\phi + F^2})y + \\
& l_7 Sinh(a + \sqrt{\phi + F^2})y + l_8 Sinh(a - \sqrt{\phi + F^2})y + l_9 y^2 Coshay + \\
& l_{10} y^2 Sinha\, y + l_{11} yCoshay + l_{12} ySinha\, y + l_{13} y^2 Cosh(\sqrt{\phi + F^2})y + \\
& l_{14} y^2 Sinh(\sqrt{\phi + F^2})y + l_{15} yCosh(\sqrt{\phi + F^2})y + l_{16} ySin(\sqrt{\phi + F^2})y + \\
& l_{17} y^4 + l_{18} y^3 + l_{19} y^2
\end{aligned} \qquad (29)$$

for the case of heat absorption.

The dimensionless temperature field is obtained from the equation (17) by using the velocity fields from equations (26) and (28) which is given by

$$\theta=\frac{-1}{\lambda}[-C_5(\phi-F^2)\left(\frac{(\phi-F^2)}{a^2}+1\right)Cos(\sqrt{\phi-F^2})y-$$

$$c_6(\phi-F^2)\left(\frac{(\phi-F^2)}{a^2}+1\right)Sin(\sqrt{\phi-F^2})y+\varepsilon[-C_{11}(\phi-F^2)\left(\frac{(\phi-F^2)}{a^2}+1\right)$$

$$Cos(\sqrt{\phi-F^2})y-C_{12}(\phi-F^2)\left(\frac{(\phi-F^2)}{a^2}+1\right)Sin(\sqrt{\phi-F^2})y-12l_1a^2Cosh2ay-$$

$$l_2\left(\frac{16(\phi-F^2)}{a^2}+4(\phi-F^2)\right)Cos(2\sqrt{\phi-F^2})y-12l_3a^2Sinh2ay-$$

$$l_4\left(\frac{16(\phi-F^2)}{a^2}+4(\phi-F^2)\right)Sin(2\sqrt{\phi-F^2})y-\left(\frac{l_{40}}{a^2}-l_{20}\right)CoshayCos(\sqrt{\phi-F^2})y-$$

$$\left(\frac{l_{41}}{a^2}-l_{21}\right)SinhaySin(\sqrt{\phi-F^2})y-\left(\frac{l_{32}}{a^2}-l_{22}\right)CoshaySin(\sqrt{\phi-F^2})y-$$

$$\left(\frac{l_{33}}{a^2}-l_{23}\right)SinhayCos(\sqrt{\phi-F^2})y-\left(\frac{l_{42}}{a^2}-l_{30}\right)yCoshay-$$

$$\left(\frac{l_{43}}{a^2}-l_{31}\right)ySinhay-\left(\frac{l_{34}}{a^2}-l_{24}\right)Coshay-\left(\frac{l_{35}}{a^2}-l_{25}\right)Sinhay-$$

$$\left(\frac{l_{13}(\phi-F^2)^2}{a^2}+l_{13}(\phi-F^2)\right)y^2Cos(\sqrt{\phi-F^2})y-$$

$$\left(\frac{l_{14}(\phi-F^2)^2}{a^2}+l_{14}(\phi-F^2)\right)y^2Sin(\sqrt{\phi-F^2})y-$$

$$\left(\frac{l_{36}}{a^2}-l_{26}\right)yCos(\sqrt{\phi-F^2})y-\left(\frac{l_{37}}{a^2}+l_{27}\right)ySin(\sqrt{\phi-F^2})y-$$

$$\left(\frac{l_{38}}{a^2}-l_{28}\right)Cos(\sqrt{\phi-F^2})y-\left(\frac{l_{39}}{a^2}-l_{29}\right)Sin(\sqrt{\phi-F^2})y$$

$$-\frac{24l_{17}}{a^2}+12l_{17}y^2+6l_{18}y+2l_{19}]] \tag{30}$$

for the case of heat generation and similarly for the case of heat absorption.

Isoflux-Isothermal case (Q_1-T_2)

For this case, the thermal boundary conditions for the channel walls can be written in the non-dimensional form as

$$q_1=-K\frac{dT}{dY}\text{ at }Y=-\frac{L}{2}\text{ and }T=T_2\text{ at }Y=\frac{L}{2} \tag{31}$$

The dimensionless form of above equation can be obtained by using the non-dimensional terms from the equation (10) with

$\Delta T = q_1 D / K$, gives

$$\frac{d\theta}{dy} = -1 \quad \text{at} \quad y = -\frac{1}{4} \quad \text{and} \quad \theta = R_{qt} \quad \text{at} \quad y = \frac{1}{4} \tag{32}$$

where $R_{qt} = (T_2 - T_0)/\Delta T$ is the thermal ratio parameter. Other than the no-slip conditions at the channel walls, two more boundary conditions in terms of U are needed to solve equation (7). These are induced by the conditions given in equation (32) and are obtained from equation (2) as follows.

Differentiating equation (2) with respect to Y , with $dP/dX = A$ gives

$$\frac{d^3U}{dY^3} - \frac{\eta}{\mu}\frac{d^5U}{dY^5} + \frac{\beta g}{\gamma}\frac{dT}{dY} = 0 \tag{33}$$

Equation (33) is non-dimensionalized by using equation (10) to give

$$\frac{d^3u}{dy^3} - \frac{1}{a^2}\frac{d^5u}{dy^5} + \lambda\frac{d\theta}{dy} = 0 \tag{34}$$

Evaluating the above equation at the left wall $(y = -1/4)$ yields

$$\frac{d^3u}{dy^3} - \frac{1}{a^2}\frac{d^5u}{dy^5} = \lambda \tag{35}$$

The other boundary condition at the right wall can be shown to be the same as that given for the isothermal-isothermal case with R_T replaced by R_{qt} such that

$$\frac{d^4u}{dy^4} = 48a^2 + \frac{a^2\lambda R_T}{2} \quad at \quad y = \frac{1}{4} \tag{36}$$

The integrating constants appeared in equations (26) to (35) are evaluated using the boundary conditions from equations (22) to (25) and (35).

Isothermal-Isoflux case (T_1-Q_2)

For this case the thermal boundary conditions are given by

$$q_2 = -K\frac{dT}{dY} \quad \text{at} \quad Y = \frac{Y}{2} \quad \text{and} \quad T = T_1 \quad \text{at} \quad Y = -\frac{Y}{2} \tag{37}$$

The dimensionless form of equation (37) can be obtained by using equation (10) with $\Delta T = q_2 D / K$ to give

$$\frac{d\theta}{dy} = -1 \quad \text{at} \quad y = \frac{1}{4} \quad \text{and} \quad \theta = R_{tq} \quad \text{at} \quad y = -\frac{1}{4} \tag{38}$$

where $R_{tq} = (T_1 - T_0)/\Delta T$ is the thermal ratio parameter for the isothermal-isoflux case. Similar to the procedure done in the previous section on isoflux-isothermal walls, the dimensionless form of the boundary conditions obtained from equation (2) and applying equation (38) can be written as

$$\frac{d^3u}{dy^3}-\frac{1}{a^2}\frac{d^5u}{dy^5}=\lambda \text{ at } y=\frac{1}{4} \quad (39)$$

The other boundary condition at the right wall can be shown to be the same as that given for the isothermal-isothermal case with R_T replaced by R_{tq} such that

$$\frac{d^4u}{dy^4}=48a^2-\frac{a^2\lambda R_T}{2} \quad at \quad y=-\frac{1}{4} \quad (40)$$

Using these boundary conditions, the integrating constants appeared in equations (20), (21) and (18)are evaluated.

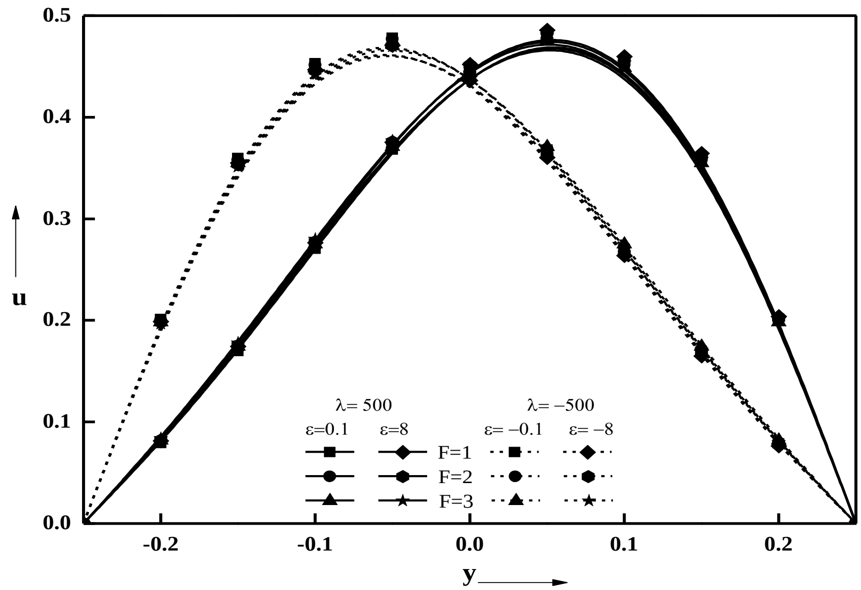
Results and Discussion

The theory of couple stress fluid due to Stokes is used to formulate a set of boundary layer equations for a flow of incompressible, couple stress fluid in a vertical channel for mixed convection. Analytical solutions are obtained using perturbation technique valid for different values of and radiation parameter F.

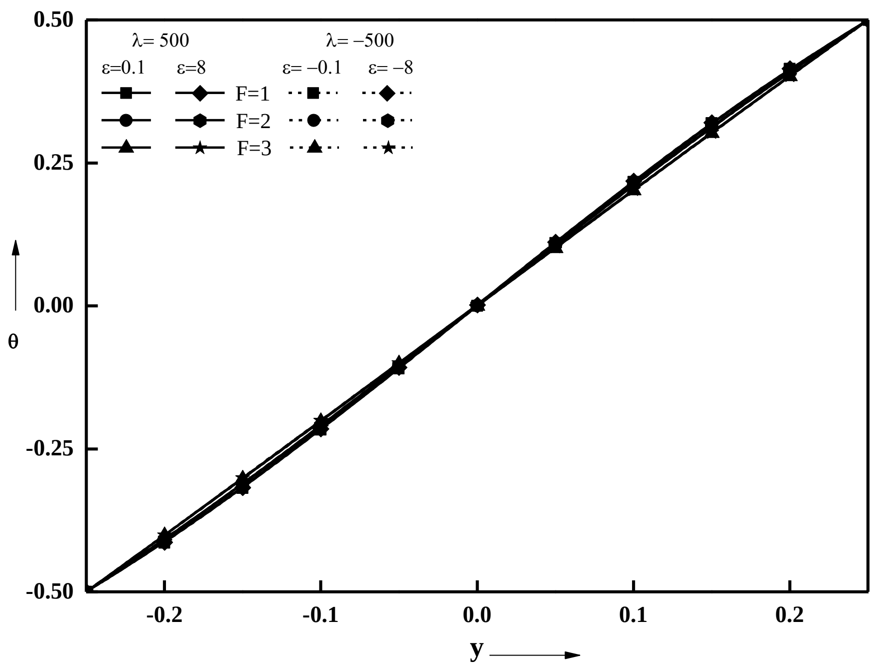
The effect of radiation parameter F on velocity and temperature is shown in Graphs 1 and 2. It is observed that as F increases, velocity decreases for upward and downward flow and the effect is not significant. The effect of couple stress parameter on temperature is not much sensible for different values of F and it almost varies linearly.Graphs 3 and 4 shows the effect of and on velocity and temperature. When the flow is upward, and are positive and on the other hand, the flow is downward when and are negative. The effect of the radiation parameter F on the flow for couple stress fluid is dominating compared to viscous fluid both on velocity and temperature. The profile of temperature are significant for couple stress fluid for different

Graph 5 shows the velocity for various valuesof radiation parameter F for . It shows that for positive values of , the fluid flow is suppressed with the reversal flow near the left wall. The same effect is observed for negative values of also. Graph 6 displays the effect of temperature field for different values of F. The effect is observed to be not sensible.Graphs 7 and 8 displays velocity and temperature profiles for Isoflux-Isothermal wall heating conditions for Rqt=1 for different values of radiation parameter F. It is seen that as radiation parameter F increases, velocity decreases for positive and is assisted for negative .The temperature profile also decreases as F increases and the magnitude of suppression is observed accordingly.

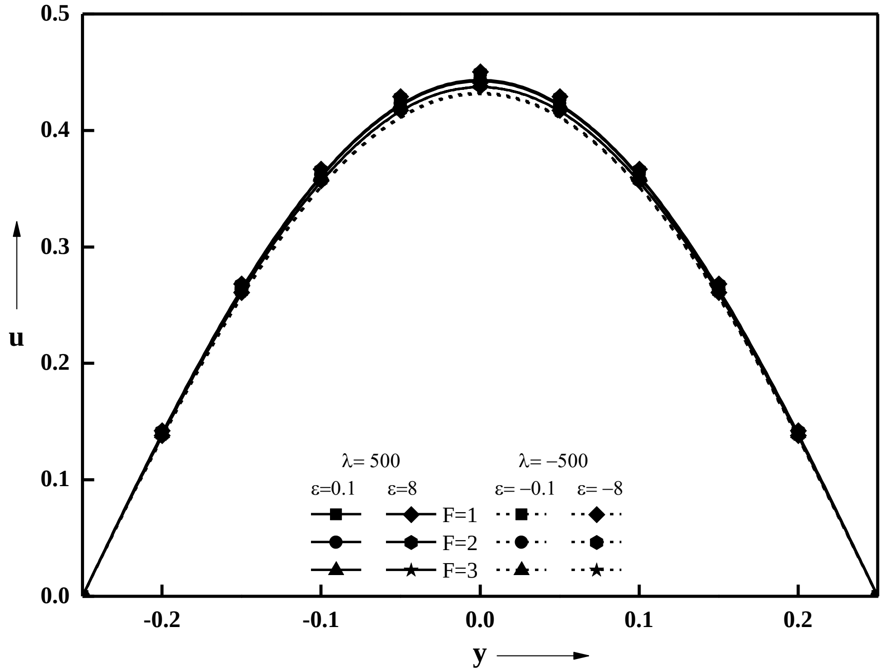
Graphs 9 and 10 are the profiles for velocity and temperature respectively for different values of and . As the radiation parameter F increases, flow is suppressed for open circuit and flow is dominating for closed circuit. The temperature profiles are not very sensible for different values of . As the radiation parameter F increases, the temperature varies linearly.



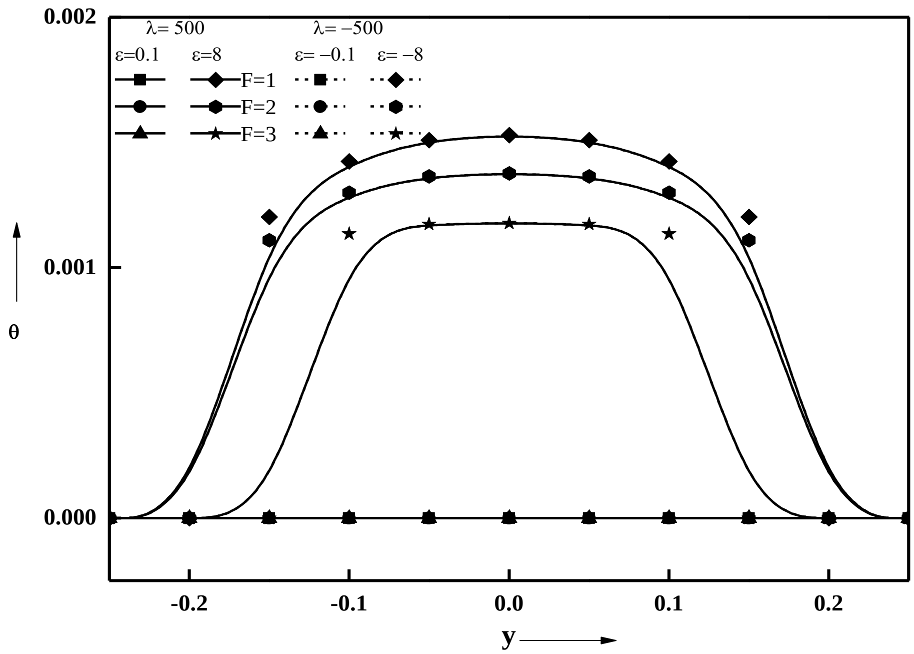
Graph 1 Plots of u / y in the case of asymmetric heating for different values of ε and F with heat generation coefficient ϕ =10 and a=4



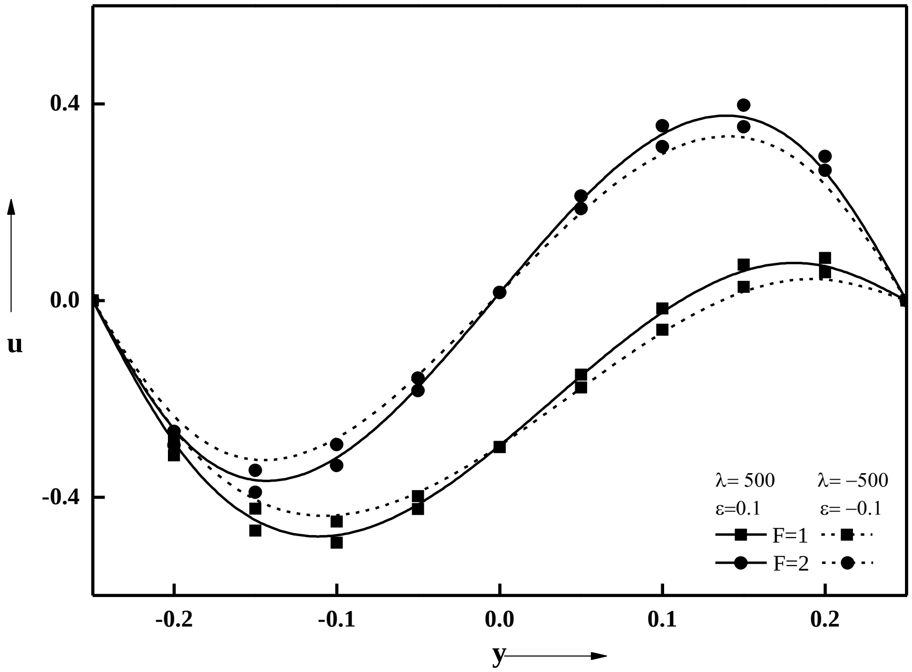
Graph 2 Plots of θ / y in the case of asymmetric heating for different values of ε and F with heat generation coefficient ϕ =10 and a=4



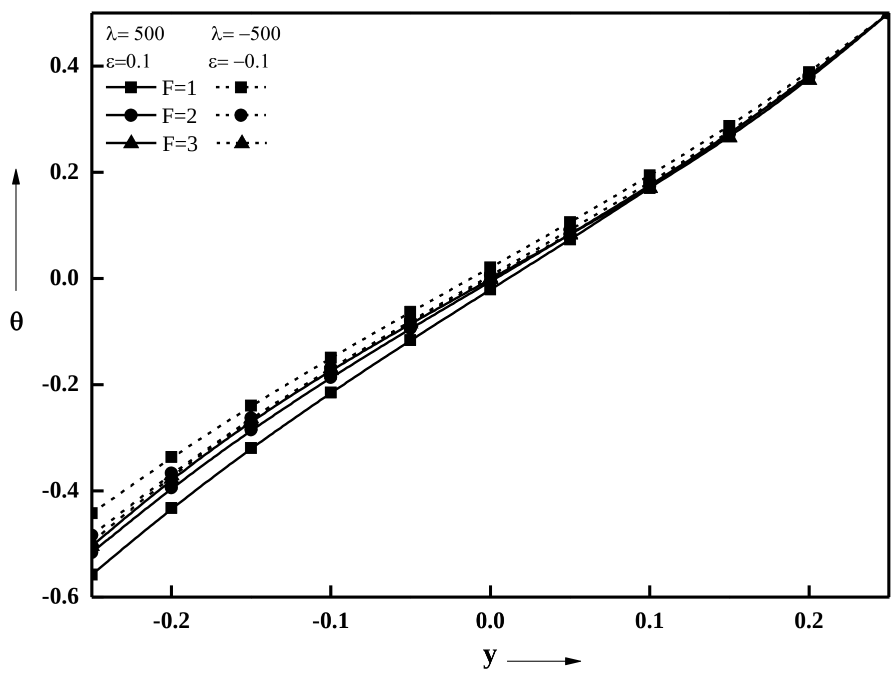
Graph 3 Plots of u / y in the case of symmetric heating for different values of ε and F with heat generation coefficient ϕ =10 and a=4.



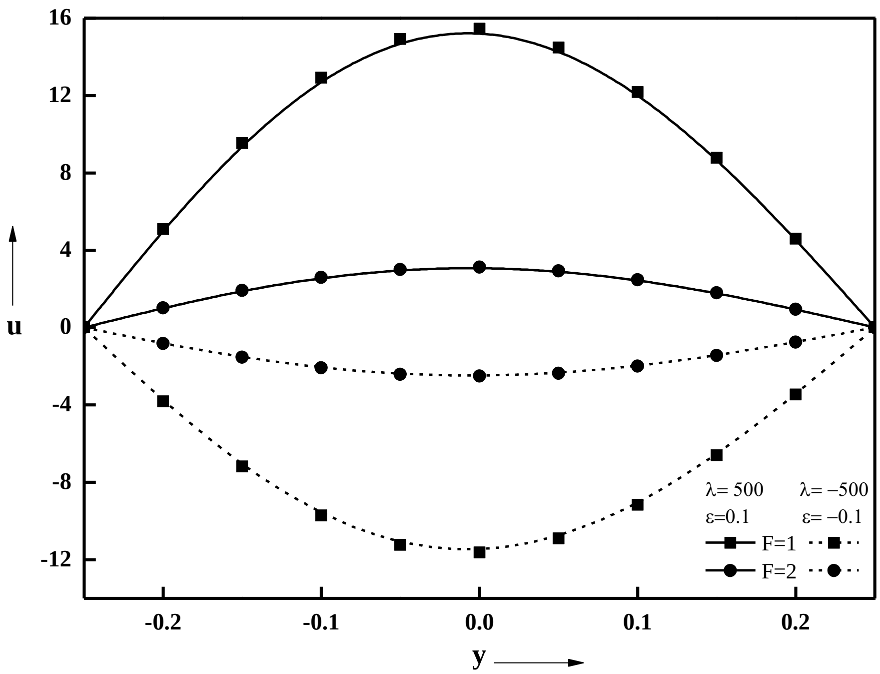
Graph 4 Plots of θ / y in the case of symmetric heating for different values of ε and F with heat generation coefficient ϕ =10 and a=4



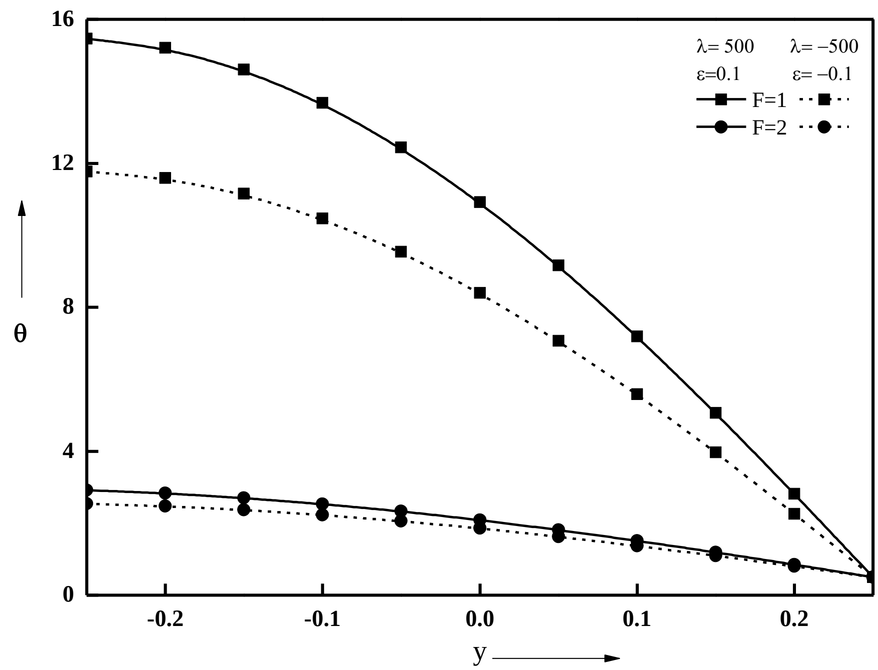
Graph 5 Plots of u / y in the case of asymmetric heating for different values of ε and F with heat absorption coefficient ϕ =10 and a=2



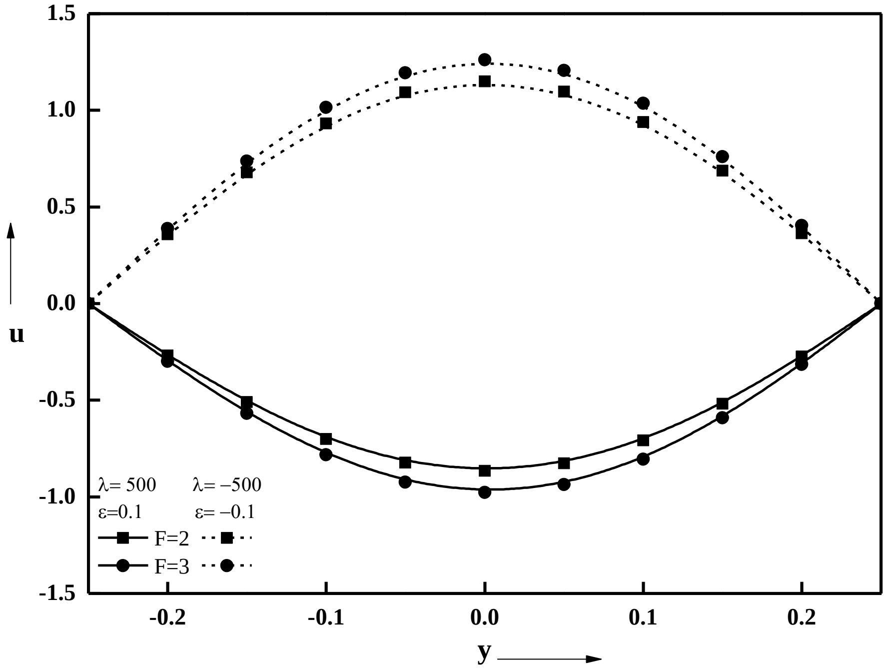
Graph 6 Plots of θ / y in the case of asymmetric heating for different values of ε and F with heat absorption coefficient ϕ =10 and a=2



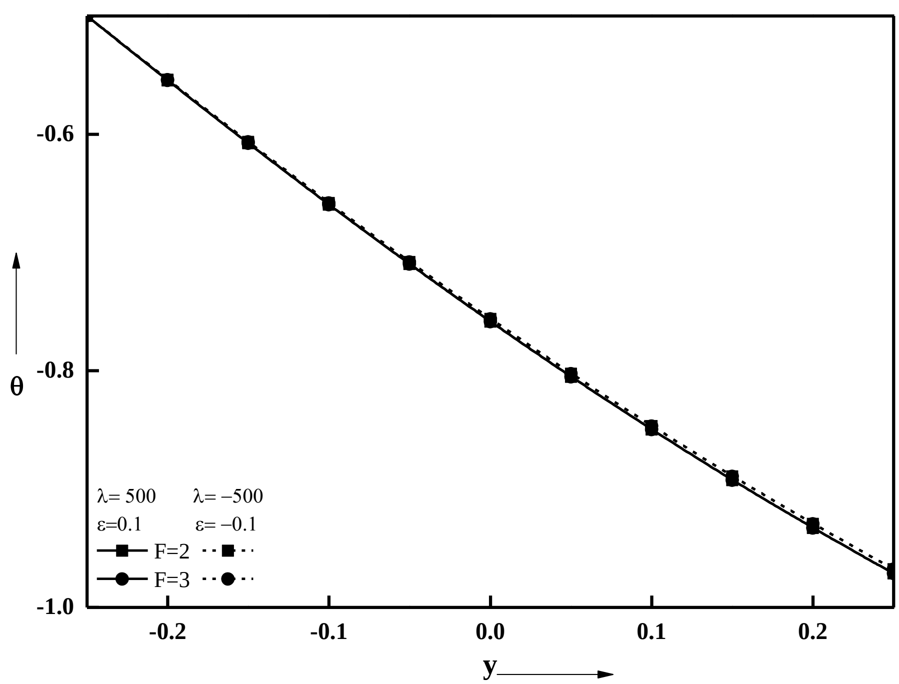
Graph 7 Plots of u / y for different values of ε and F for Isoflux-Isothermal case with heat generation coefficient ϕ =10 and a=2



Graph 8 Plots of θ / y for different values of ε and F for Isoflux-Isothermal case with heat generation coefficient ϕ =10 and a=2



Graph 9 Plots of u / y for different values of ε and F for Isothermal-Isoflux case with heat generation coefficient ϕ =10 and a=2



Graph 10 Plots of θ / y for different values of ε and F for Isothermal-Isoflux case with heat generation coefficient ϕ =10 and a=4

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