

INVESTIGATION OF N-POLICY MARKOVIAN QUEUEING SYSTEM WITH DESPONDENT ARRIVALS, RETAINMENT OF RENEGED CUSTOMERS AND CONTROLLABLE ARRIVAL RATES

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Abstract

Afinite capacity, finite source, Markovian queueing model with despondent arrivals, retainment of renegeed customers and controllable arrival rates is considered. Using this model, a steady-state solution and system characteristics can be obtained. The measures of effectiveness of the queueing model are also derived. Numerical results are given for better understanding and relevant conclusion is presented.

Keywords: Infinite capacity, finite source, retainment of renegeed customers, despondent arrivals, single server, interdependent, bivariate Poisson process.

1. INTRODUCTION

Some researcher worker have analysed the N-policy for queues. N-policy denotes that the server stay idle until there are N-customers hold back in the queue. Service starts with the arrival of the Nth customer and the busy period continues till the system is empty. A customer may enter the queue, but after a time lose patience and decide to leave the queue after waiting for a long time. In this case, he is said to have renegeed. Buyer who tend to be irritated may not always be deprived of courage by immoderate queue size, but may instead join the queue to see how long they have to wait, all the time retaining the exclusive to renege if their evaluation of their waiting is unbearable.

Queueing system with discouraged arrivals have applications in computers with lot of job processing where job submissions are discouraged when the system is used often and arrivals are patterned as a poisson process with state dependent arrival rate. The analytical rules that are required to study the probability theory in queue has been explained in [8] The retainment affects the arrival rate of the queueing system has been discussed. [1] have studied M/M/1/N queueing system with balking and renegeing and performed its steady state analysis. Most desirable working rule reached in the process of total costs associated and the uncertainty in the output of a model is presented in [2]. [11] described single server finite capacity queueing system with inverse balking. [5] discussed M/M/1/N queueing model for modeling supply chain situations facing customer abandonment. [13] discussed multiserver queue with balking, renegeing and more server. [15] analysed a single server queue with impatient customers and the situations of abandoned customers have been discussed. [12] studied limited manageable queueing system with balking and renegeing. [19] considered discouragement in which the advent rate decreases according to a negative exponential law. Busy time investigation of a correspondent queues discussed in [6]. Correlation between queueing models with balking and renegeing and machine repair problem with spares obtained in [9]. Controlling incoming and outgoing rates to decrease sensitivity of queueing systems with impatient customers studied in [14].

A group arrival customers with more servers under N-policy have analysed in [7]. Making rational decision while staying in the queue and the probable effect of this decision studied in [10]. Modified N-policy for Markovian general service queues obtained

in [16]. Queueing models where moving customers are discouraged by queue length are discussed by many researchers in their research work. [23] discussed multi-server model with discouragement.

Much work has been reported in literature regarding interdependent queueing model with controllable arrival rates. [21] derived interdependent Queueing model with jockeying. [24,-27] discussed various interdependent queueing model with controllable arrival rates. [22] obtained a single server Markovian queueing system with discouraged arrivals and retention of renege Customers. [20] studied single unreliable server interdependent loss and delay queueing model with controllable arrival rate under N-policy. [3,4] have extended the work to controllable arrival rates in retrial queue.

A finite capacity, finite source retention of renege customers queueing system with N- policy is considered with the assumption that the arrival and service processes were associated and act in accordance with bivariate poisson process. Inclusive of this mutually dependent, the system has two arrivals rates, - higher arrival rate and - lower arrival rate which controlled the incoming arrivals.

2. MODEL DESCRIPTION

Consider a single server, infinite capacity, dependent arrivals and retention of renege customers queueing model where arrivals occurs according to the poisson flow of rate and are determined by the number of customers in the system and outgoing times are independently, identically and exponentially distributed with rate .

A renegeing concept, which means that a unit after being in the queue for a service a certain time may leave the queue without being served. The renegeing times follow exponential distribution with parameters and stay in the queue for further service with probability and he may abandon the queue without receiving the service with a probability . The arrival process and the service process of the system are correlated and follow a bivariate Poisson process given by

$$P[Z_1(t) = z_1 \quad Z_2(t) = z_2] = e^{-(\lambda_i + \mu - \epsilon)t} \sum_{j=0}^{\min(z_1, z_2)} \frac{(\epsilon t)^j [(\lambda_i - \epsilon)t]^{z_1 - j} [(\mu - \epsilon)t]^{z_2 - j}}{j! (z_1 - j)! (z_2 - j)!}$$

where $z_1, z_2 = 0, 1, 2, \dots$, $\lambda_i > 0, i = 0, 1$; $\mu > 0$, $0 \leq \epsilon < \min(\lambda_i, \mu), i = 0, 1$,

with parameters and as mean higher rate of incoming arrivals, mean lower rate of incoming arrivals, mean outgoing customer's rate and mean dependence rate

3. STEADY STATE EQUATIONS

= The probability in which n consumers in the line if the system is experiencing a higher rate of arrivals and the server is unoccupied.

= The probability assuming there are n customers in the line when the system has a higher rate of arrivals, and the server is overloaded.

= The probability of n customers in the line while the system's arrival rate is decreasing and the server is busy.

We may see that exist for, exist for and exist for

The Steady state equations are

$$-(M-n)(\lambda_1-\epsilon)P_{0,n}(0) + (M-n+1)(\lambda_1-\epsilon)P_{0,n-1}(0) = 0, \quad 1 \leq n \leq N-1 \dots\dots\dots(1)$$

$$-M(\lambda_1-\epsilon)P_{0,0}(0) + (\mu-\epsilon)P_{1,1}(0) = 0 \dots\dots\dots(2)$$

$$-\left[(M-2)\left(\frac{\lambda_1-\epsilon}{n+1}\right) + (\mu-\epsilon) \right] P_{1,1}(0) + [(\mu-\epsilon) + \alpha\beta] P_{1,2}(0) = 0 \dots\dots\dots(3)$$

$$-\left[(M-n-1)\left(\frac{\lambda_1-\epsilon}{n+1}\right) + (\mu-\epsilon) + (n-1)\alpha\beta \right] P_{1,n}(0) + [(\mu-\epsilon) + n\alpha\beta] P_{1,n+1}(0) \\ + (M-n)\left(\frac{\lambda_1-\epsilon}{n}\right) P_{1,n-1}(0) = 0, \quad 2 \leq n \leq N-1 \dots\dots\dots(4)$$

$$-\left[(M-N-1)\left(\frac{\lambda_1-\epsilon}{N+1}\right) + (\mu-\epsilon) + (N-1)\alpha\beta \right] P_{1,N}(0) + (M-N)\left(\frac{\lambda_1-\epsilon}{N}\right) P_{0,N-1}(0) \\ + [(\mu-\epsilon) + n\alpha\beta] P_{1,N+1}(0) + (M-N)\left(\frac{\lambda_1-\epsilon}{N}\right) P_{1,N-1}(0) = 0 \dots\dots\dots(5)$$

$$-\left[(M-n-1)\left(\frac{\lambda_1-\epsilon}{n+1}\right) + (\mu-\epsilon) + (n-1)\alpha\beta \right] P_{1,n}(0) + [(\mu-\epsilon) + n\alpha\beta] P_{1,n+1}(0) \\ + (M-n)\left(\frac{\lambda_1-\epsilon}{n}\right) P_{1,n-1}(0) = 0, \quad n=N+1, N+2, N+3 \dots r-1 \dots\dots\dots(6)$$

$$-\left[(M-r-1)\left(\frac{\lambda_0-\epsilon}{r+1}\right) + (\mu-\epsilon) + (r-1)\alpha\beta \right] P_{1,r}(0) + [(\mu-\epsilon) + r\alpha\beta] P_{1,r+1}(0) \\ + (M-r)\left(\frac{\lambda_0-\epsilon}{r}\right) P_{1,r-1}(0) + [(\mu-\epsilon) + r\alpha\beta] P_{1,r+1}(1) = 0 \dots\dots\dots(7)$$

$$-\left[(M-n-1)\left(\frac{\lambda_1-\epsilon}{n+1}\right) + (\mu-\epsilon) + (n-1)\alpha\beta \right] P_{1,n}(0) + [(\mu-\epsilon) + n\alpha\beta] P_{n+1}(0) \\ + (M-n+1)\left(\frac{\lambda_1-\epsilon}{n}\right) P_{n-1}(0) = 0, \quad n = r+1, r+2, \dots R-2 \dots\dots\dots(8)$$

$$-\left[(M-R)\left(\frac{\lambda_1-\epsilon}{R}\right) + (\mu-\epsilon) + (R-2)\alpha\beta \right] P_{1,R-1}(0) + (M-R+1)\left(\frac{\lambda_1-\epsilon}{R-1}\right) P_{1,R-2}(0) = 0 \dots\dots\dots(9)$$

$$-\left[(M-r)\left(\frac{\lambda_2-\epsilon}{r+2}\right) + (\mu-\epsilon) + r\alpha\beta \right] P_{1,r+1}(1) \\ + [(\mu-\epsilon) + (r+1)\alpha\beta] P_{1,r+2}(1) = 0 \dots\dots\dots(10)$$

$$-\left[(M-n-1) \binom{\lambda_2 - \epsilon}{n+1} + (\mu - \epsilon) + (n-1)\alpha\beta \right] P_{1,n}(1) + [(\mu - \epsilon) + n\alpha\beta] P_{1,n+1}(1) + (M-n) \binom{\lambda_2 - \epsilon}{n} P_{1,n-1}(1) = 0 \quad n = r+2, r+3, \dots, R-1 \dots \dots \dots (11)$$

$$-\left[(M-R-1) \binom{\lambda_2 - \epsilon}{R+1} + (\mu - \epsilon) + (R-1)\alpha\beta \right] P_{1,R}(1) + [(\mu - \epsilon) + R\alpha\beta] P_{1,R+1}(1) + (M-R) \binom{\lambda_2 - \epsilon}{R} P_{1,R-1}(1) + (M-R) \binom{\lambda_1 - \epsilon}{R} P_{1,R-1}(0) = 0 \dots \dots \dots (12)$$

$$-\left[(M-n-1) \binom{\lambda_2 - \epsilon}{n+1} + (\mu - \epsilon) + (n-1)\alpha\beta \right] P_{1,n}(1) + [(\mu - \epsilon) + n\alpha\beta] P_{1,n+1}(1) + (M-n) \binom{\lambda_2 - \epsilon}{n} P_{1,n-1}(1) = 0, \quad n = R+1, R+2, \dots, K-1 \dots \dots \dots (13)$$

$$-[(\mu - \epsilon) + (K-1)\alpha\beta] P_{1,K}(1) + (M-K) \binom{\lambda_1 - \epsilon}{K} P_{1,K-1}(1) = 0 \dots \dots \dots (14)$$

From the equation (1),

$$P_{0n}(0) = 1 + \frac{(\lambda_1 - \epsilon)}{(M-n)(\lambda_1 - \epsilon)} P_{00}(0) \quad n = 1, 2, \dots, N-1 \quad (15)$$

From (2) and (4) we get

$$P_{1,n}(0) = \left[\frac{1}{n!} \prod_{s=1}^n \frac{(M-n-1)(\lambda_1 - \epsilon)}{[(\mu - \epsilon) + (s-1)\alpha\beta]} \right] P_{0,0}(0) \quad , n = 2, 3, \dots, N-1 \dots \dots \dots (16)$$

From (5) we get

$$P_{1,N}(0) = \left[\frac{1}{N!} \prod_{s=1}^N \frac{(M-N-1)(\lambda_1 - \epsilon)}{[(\mu - \epsilon) + (s-1)\alpha\beta]} \right] P_{0,0}(0) \quad \dots \dots \dots (17)$$

From (6) we get

$$P_{1,n}(0) = \left[\frac{1}{n!} \prod_{s=1}^n \frac{(M-n-1)(\lambda_1 - \epsilon)}{[(\mu - \epsilon) + (s-1)\alpha\beta]} \right] P_{0,0}(0) \quad , n = N+1, N+2, \dots, r-1 \dots \dots (18)$$

Using (7) we get

$$P_{1,r+1}(0) = \left[\frac{1}{(r+1)!} \prod_{s=1}^{r+1} \frac{(M-r)(\lambda_0-\epsilon)}{[(\mu-\epsilon)+(s-1)\alpha\beta]} \right] P_0(0) - P_{r+1}(1) \dots\dots\dots(19)$$

Using the equation (8), We recursively derive

$$P_{1,n}(0) = \left[\frac{1}{n!} \prod_{s=1}^n \frac{(M-n-1)(\lambda_1-\epsilon)}{[(\mu-\epsilon)+(k-1)\alpha\beta]} \right] P_{0,0}(0) - \frac{A_1 P_{r+1}(1)}{\prod_{l=r+1}^{n-1} [(\mu-\epsilon)+l\alpha\beta]} \dots\dots\dots(20)$$

where

$$A_1 = \frac{(\lambda_1-\epsilon)^{n-r-1}}{n P_{n-r-1}} + \frac{(\lambda_1-\epsilon)^{n-r-2}}{n P_{n-r-2}} \prod_{i=r}^{R-1} [(\mu-\epsilon)+i\alpha\beta], \quad n = r+1, r+2, \dots, R-1$$

Using the equation (9), We get,

$$P_{1,r+1}(1) = \left[\frac{\frac{(M-r)(\lambda_0-\epsilon)^R}{R!} \prod_{s=1}^{r+1} \frac{1}{[(\mu-\epsilon)+(k-1)\alpha\beta]}}{A_2} \right] P_0(0) \dots\dots\dots(21)$$

where $A_2 = \frac{(\lambda_1-\epsilon)^{R-r-1}}{R P_{R-r-1}} + \frac{(\lambda_1-\epsilon)^{R-r-2}}{R P_{R-r-2}} \prod_{i=r}^{R-2} [\mu-\epsilon+i\alpha\beta]$

Using the equation(11)we get $P_{1,n}(1) = \frac{A_3 P_{r+1}(1)}{\prod_{l=r+1}^{n-1} [(\mu-\epsilon)+l\alpha\beta]} \dots\dots\dots(22)$

where $A_3 = \frac{(M-n-1)(\lambda_2-\epsilon)^{n-r-1}}{n P_{n-r-1}} + \frac{(M-n-1)(\lambda_2-\epsilon)^{n-r-2}}{n P_{n-r-2}} \prod_{i=r+1}^{R-1} [\mu-\epsilon+i\alpha\beta]$

$$n = r+1, r+2, r+3, \dots, R-1$$

Using the equation (12),

$$P_{1,R+1}(1) = \frac{P_{r+1}(1)}{\prod_{l=r+1}^R (\mu-\epsilon)+l\alpha\beta}$$

$$= \left[\frac{(M-R)(\lambda_2 - \epsilon)^{R-r}}{(R+1)P_{R-r}} + \frac{(M-R)(\lambda_2 - \epsilon)^{R-r-1}}{(R+1)P_{R-r-1}} \prod_{i=r}^{R-2} (\mu - \epsilon + i\alpha\beta) \right] \dots\dots\dots(23)$$

where $P_{r+1}(1)$ is given by (21)

Using the equation (13), we recursively derive

$$P_{1,n}(1) = \frac{A_4 P_{r+1}(1)}{\prod_{l=r+1}^K [(\mu - \epsilon) + l\alpha\beta]}$$

where $P_{r+1}(1)$ is given by (21) and

$$A_4 = \frac{(M-n-1)(\lambda_2 - \epsilon)^{n-r-1}}{n P_{n-r-1}} + \frac{(M-n-1)(\lambda_2 - \epsilon)^{n-r-2}}{n P_{n-r-2}} (\mu - \epsilon + r\alpha\beta) + \dots + \frac{(M-n-1)(\lambda_2 - \epsilon)^{K-R}}{n P_{n-R}} \prod_{i=r}^{R-2} (\mu - \epsilon + i\alpha\beta) \dots\dots\dots(24)$$

$n = R+1, R+2, R+3, \dots, K-1$

4. CHARACTERISTICS OF THE MODEL

The probability that the system is in higher rate of arrivals is

$$P(0) = \sum_{n=0}^r P_n(0) + \sum_{n=r+1}^{R-1} P_n(0) \dots\dots\dots(25)$$

Since $P_n(0)$ exist only when $0 \leq n \leq R-1$

From (15) to (21) we get

$$P(0) = \sum_{n=0}^{R-1} \left[\frac{(M-n-1)(\lambda_1 - \epsilon)^n}{n!} \prod_{s=1}^n \frac{1}{[(\mu - \epsilon) + (s-1)\alpha\beta]} \right] P_{0,0}(0) - \sum_{n=r+1}^{R-1} \left[\frac{(M-n-1)(\lambda_1 - \epsilon)^R}{R!} \prod_{s=1}^{R-1} \left[\frac{1}{[(\mu - \epsilon) + (s-1)\alpha\beta]} \right] \frac{A_1}{A_2} P_{0,0}(0) \right] \dots\dots\dots(26)$$

where A_1 and A_2 is given by (20) and (21)

The Probability that the system is in lower rate of arrival is

$$P(1) = \sum_{n=r+1}^R P_n(1) + \sum_{n=R+1}^K P_n(1)$$

Since $P_n(1)$ exists only when $r+1 \leq n \leq K$

$$P(1) = \sum_{n=r+1}^R \left[\frac{(M-n-1)(\lambda_2-\epsilon)^R}{R!} \prod_{l=1}^{n-1} \frac{1}{[(\mu-\epsilon)+l\alpha\beta]} \right] \frac{A_3}{A_2} P_0(0) + \sum_{n=R+1}^K \left[\frac{(M-n-1)(\lambda_2-\epsilon)^R}{R!} \prod_{l=1}^{k-1} \frac{1}{[(\mu-\epsilon)+l\alpha\beta]} \right] \frac{A_4}{A_2} P_0(0) \dots \dots \dots (27)$$

where A_3 and A_4 is given by (22) and (23)

The probability $[P_0(0)]$ that the system is empty can be calculated from the normalizing condition.

$$P(0) + P(1) = 1$$

From (26) and (27) we get

$$P_0(0) = \frac{1}{\left\{ 1 + \sum_{n=1}^{R-1} \left[\frac{(M-n-1)(\lambda_0-\epsilon)^n}{n!} \prod_{s=1}^n \frac{1}{[(\mu-\epsilon)+(s-1)\alpha\beta]} \right] - \sum_{n=r+1}^{R-1} \frac{(M-n-1)(\lambda_0-\epsilon)^R}{R!} \prod_{s=1}^{r+1} \left(\frac{1}{[(\mu-\epsilon)+(s-1)\alpha\beta]} \right) \frac{A_1}{A_2} + \sum_{n=r+1}^R \frac{(M-n-1)(\lambda_0-\epsilon)^R}{R!} \prod_{l=1}^{n-1} \left(\frac{1}{[(\mu-\epsilon)+l\alpha\beta]} \right) \frac{A_3}{A_2} + \sum_{n=R+1}^K \frac{(M-n-1)(\lambda_0-\epsilon)^R}{R!} \prod_{l=1}^{K-1} \left(\frac{1}{[(\mu-\epsilon)+l\alpha\beta]} \right) \frac{A_4}{A_2} \right\}} \dots \dots \dots (28)$$

where A_1, A_2, A_3 and A_4 is given by (20) (21) (22) and (24).

The average number of customers in the system, is given by

$$L_s = L_{s_0} + L_{s_1}$$

where

$$L_{s_0} = \sum_{n=0}^r n P_n(0) + \sum_{n=r+1}^{R-1} n P_n(0)$$

$$\text{and } L_{S_1} = \sum_{n=r+1}^{R-1} nP_n(1) + \sum_{n=R}^K nP_n(1)$$

$$L_{S_0} = \sum_{n=0}^{r-1} \left[\frac{1}{n!} \prod_{s=1}^n \frac{(M-n-1)(\lambda_0-\epsilon)}{[(\mu-\epsilon)+(s-1)\alpha\beta]} \right] P_0(0) - \sum_{n=r+1}^{R-1} \left(\frac{(M-n-1)(\lambda_0-\epsilon)^R}{R!} \prod_{s=1}^{r+1} \frac{1}{[(\mu-\epsilon)+(s-1)\alpha\beta]} \right) \frac{A_1}{A_2} P_0(0)$$

where A_1 , A_2 and $P_0(0)$ is given by (20) (21) and (28) and

$$L_{S_1} = \sum_{n=r+1}^{R-1} \left[n \frac{(M-n-1)(\lambda_2-\epsilon)^R}{R!} \prod_{t=1}^{n-1} \left(\frac{1}{[(\mu-\epsilon)+t\alpha\beta]} \right) \right] \frac{A_3}{A_2} P_0(0) + \sum_{n=R}^K \left[n \frac{(M-n-1)(\lambda_2-\epsilon)^R}{R!} \prod_{t=1}^K \left(\frac{1}{[(\mu-\epsilon)+t\alpha\beta]} \right) \right] \frac{A_4}{A_2} P_0(0)$$

where A_2 , A_3 , A_4 and $P_0(0)$ is given by (21) (22) (24) and (28)

Using Little's Formula the expected waiting time of the customers in the system is given as $W_s = \frac{L_s}{\lambda}$

where $\bar{\lambda} = \lambda_1 P(0) + \lambda_2 P(1)$

5. NUMERICAL ILLUSTRATIONS

For several values of $\lambda_1, \lambda_2, \mu, r, R, \epsilon, \alpha, \beta, K, N, M$ the values of $P_0(0), P(0), P(1), L_s$ and W_s are determined and listed in the following Table.

TABLE 5.1

r	R	K	λ_1	λ_2	μ	ϵ	β	α	N	M	$P_0(0)$	P(0)	P(1)	L_s	W_s
3	5	9	4	3	5	0	0.1	0.1	3	11	0.434	0.98615	0.01384	1.0680	0.2741
3	5	9	4	3	5	0.5	0.1	0.1	3	11	0.445	0.96915	0.03085	5.4361	1.6202
3	5	9	4	3	5	0	0.1	0.1	3	11	0.478	0.98846	0.01155	0.7660	0.1766
3	5	9	4	3	6	0.5	0.1	0.1	3	11	0.545	0.99431	0.00569	0.5835	0.1384

3	5	9	4	3	8	0.5	0.1	0.1	3	11	0.625	0.99579	0.00421	0.4388	0.1227
3	5	9	4	4	5	0.5	0.1	0.1	3	11	0.462	0.99319	0.00681	0.6971	0.1671
3	5	9	4	4	5	0	0.1	0.1	3	11	0.475	0.98815	0.00252	0.7305	0.1879
3	5	9	4	3	5	0.5	0.1	0.1	3	11	0.493	0.99417	0.00583	0.7057	0.1765
3	5	9	3	2	5	0.5	0.1	0.1	3	11	0.575	0.99714	0.00286	0.5133	0.1705
3	5	9	2	1	5	0.5	0.1	0.1	3	11	0.645	0.99981	0.00019	0.3346	0.1563
3	5	9	6	3	5	0.5	0.1	0.1	3	11	0.332	0.95454	0.04546	1.1471	0.1843
3	5	9	6	5	4	0	0.1	0.2	3	11	0.252	0.93541	0.06459	1.4593	0.2532
3	5	9	5	5	5	0	0.1	0.1	3	11	0.382	0.97159	0.02841	0.9467	0.1814
3	5	9	4	3	5	1	0.1	0.1	3	11	0.453	0.99921	0.00079	0.5732	0.1424
3	5	9	4	3	6	1	0.2	0.2	3	11	0.581	0.99851	0.00149	0.5307	0.1301
3	5	9	4	3	6	0.5	0.2	0.1	3	13	0.478	0.98734	0.01266	0.5436	0.1485
3	5	9	4	4	5	0.5	0.2	0.1	3	14	0.508	0.98611	0.01389	0.6888	0.1462

6. CONCLUSIONS

Numerical results for finite source, single server controllable arrival rates, and retainment of renege customers in interdependent queueing model is given in the above table. When the arrival rate decreases, LS and WS decrease. The queue length and waiting time decrease, when the servers and service rate increases. While all other parameters remains constant, LS and WS decreases as the average dependent rate goes up. Ls and Ws increase as the arrival rate rises while the other variables are fixed. Ls and Ws decreases when the service rate reach higher value while the other remains the same.

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